

5. The functions f and g are defined by

$$f(x) = \frac{kx}{2x-1} \quad x \in \mathbb{R} \quad x \neq \frac{1}{2}$$

$$g(x) = 2 + 3x - x^2 \quad x \in \mathbb{R}$$

where k is a non-zero constant.

(a) Find in terms of k

(i) $fg(4)$

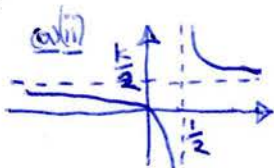
$$(a)(i) g(4) = 2 + 3(4) - 4^2 = -2$$

$$fg(4) = f(g(4)) = f(-2) \quad (1 \text{ mark})$$

$$f(-2) = \frac{k(-2)}{2(-2)-1} = \frac{-2k}{-5} = \frac{2k}{5} \quad (1 \text{ mark})$$

(ii) the range of f

(iii) f^{-1}



(a)(ii) cotd. as $x \rightarrow +\infty, x \rightarrow -\infty$, $f(x) \rightarrow \frac{k}{2}$, but it never gets there so range is $f(x) \in \mathbb{R} \quad f(x) \neq \frac{k}{2}$ (1 mark)

(6)

Given that

$$f^{-1}(2) = \frac{11}{3g(2)}$$

(b) find the exact value of k

(2)

(a)(iii) Let $y = \frac{kx}{2x-1}$ & make x the subject

$$y(2x-1) = kx$$

$$2yx - y = kx$$

$$2yx - kx = y$$

$$x(2y-k) = y \quad (1 \text{ mark})$$

$$x = \frac{y}{2y-k} \Rightarrow f^{-1}(x) = \frac{x}{2x-k} \quad (1 \text{ mark})$$

Domain of f^{-1} is $x \neq \frac{k}{2}$ because that would allow division by 0 (and domain of f^{-1} = range of f) (1 mark)

$$(b) f^{-1}(2) = \frac{2}{2(2)-k} = \frac{2}{4-k}$$

$$\frac{2}{4-k} = \frac{11}{12} \quad (1 \text{ mark})$$

$$\frac{11}{3g(2)} = \frac{11}{3(2+3(2)-2^2)} = \frac{11}{12}$$

$$2(12) = 11(4-k)$$

$$24 = 44 - 11k$$

$$\Rightarrow k = \frac{20}{11} \quad (1 \text{ mark})$$