

6.

$$f(x) = \frac{10}{\sqrt{4-3x}}$$

(a) Show that the first 4 terms in the binomial expansion of  $f(x)$ , in ascending powers of  $x$ , are

$$A + Bx + Cx^2 + \frac{675}{1024}x^3$$

where  $A, B$  and  $C$  are constants to be found. Give each constant in simplest form.

(4)

Given that this expansion is valid for  $|x| < k$

(b) expansion is valid for  $|-\frac{3}{4}x| < 1$

(b) state the largest value of  $k$ .

$$\frac{3}{4}|x| < 1 \Rightarrow |x| < \frac{4}{3}$$

$k = \frac{4}{3}$  (1 mark)

(1)

By substituting  $x = \frac{1}{3}$  into  $f(x)$  and into the answer for part (a),

(c) find an approximation for  $\sqrt{3}$

Give your answer in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers to be found.

(2)

(a)  $f(x) = 10(4-3x)^{-\frac{1}{2}} = 10(4 \times (1 - \frac{3}{4}x))^{-\frac{1}{2}}$   
 $= 10(4^{-\frac{1}{2}} \times (1 - \frac{3}{4}x)^{-\frac{1}{2}}) = 10(\frac{1}{2})(1 - \frac{3}{4}x)^{-\frac{1}{2}}$   
 $= 5(1 - \frac{3}{4}x)^{-\frac{1}{2}}$  (1 mark)

this can now be expanded

$$= 5 \left( 1 + \frac{(-\frac{1}{2})(-\frac{3}{4}x)}{1} + \frac{(-\frac{1}{2})(-\frac{1}{2}-1)}{2!} \left(-\frac{3}{4}x\right)^2 + \frac{(-\frac{1}{2})(-\frac{1}{2}-1)(-\frac{1}{2}-1-1)}{3!} \left(-\frac{3}{4}x\right)^3 + \dots \right)$$

(2 marks)

$$= 5 \left( 1 + (-\frac{1}{2})(-\frac{3}{4}x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2} \left(\frac{9}{16}x^2\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{6} \left(-\frac{27}{64}x^3\right) + \dots \right)$$

$$= 5 \left( 1 + \frac{3}{8}x + \frac{27}{128}x^2 + \frac{405}{2304}x^3 + \dots \right)$$

$$= 5 + \frac{15}{8}x + \frac{135}{128}x^2 + \frac{675}{1024}x^3 + \dots$$
 (1 mark)

(c) with  $x = \frac{1}{3}$ ,  $f(x) \approx 5 + \frac{15}{8}(\frac{1}{3}) + \frac{135}{128}(\frac{1}{3})^2 + \frac{675}{1024}(\frac{1}{3})^3 = \frac{5905}{1024}$  (1 mark)

$$f\left(\frac{1}{3}\right) = \frac{10}{\sqrt{4-3(\frac{1}{3})}} = \frac{10}{\sqrt{3}} \quad \frac{10}{\sqrt{3}} \approx \frac{5905}{1024} \Rightarrow \sqrt{3} \approx \frac{10 \times 1024}{5905} = \frac{2048}{1181}$$

(1 mark)