

7. Curve C has equation

$$y = (x^2 - 5x + 8)e^{x^2} \quad x \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = (2x^3 - 10x^2 + 18x - 5)e^{x^2} \quad (3)$$

Given that

- C has only one stationary point
- the stationary point has x coordinate α
- $\frac{dy}{dx} \approx -0.5$ at $x = 0.3$
- $\frac{dy}{dx} \approx 0.9$ at $x = 0.4$

$y = uv$ where $u = x^2 - 5x + 8$
 $v = e^{x^2}$
 by Product Rule, $y' = uv' + u'v$
 $u' = 2x - 5$, $v' = e^{x^2} \times 2x$
 by Chain Rule
 $= 2xe^{x^2}$
 so, $y' = (x^2 - 5x + 8)2xe^{x^2} + (2x - 5)e^{x^2}$ (1 mark)
 $= (2x^3 - 10x^2 + 16x)e^{x^2} + (2x - 5)e^{x^2}$
 $= (2x^3 - 10x^2 + 18x - 5)e^{x^2}$ (1 mark)

(b) explain why $0.3 < \alpha < 0.4$

(b) there is a sign change from $x=0.3$ to $x=0.4$ & function is continuous, so $0.3 < \alpha < 0.4$ (1 mark)

(c) Show that α is a solution of the equation

(c) $e^{x^2} > 0$, so $\frac{dy}{dx} = 0$
 $\Rightarrow 2x^3 - 10x^2 + 18x - 5 = 0$ (1 mark)
 $x = \frac{5(2x^2 + 1)}{2(x^2 + 9)}$ (3)

(d) Using the iteration formula

$$x_{n+1} = \frac{5(2x_n^2 + 1)}{2(x_n^2 + 9)} \quad \text{with } x_1 = 0.3$$

find

- (i) the value of x_3 to 4 decimal places,
- (ii) the value of α to 4 decimal places.

(3)

(c) contd. $2x^3 + 18x = 10x^2 + 5$
 $2x(x^2 + 9) = 5(2x^2 + 1)$
 $x = \frac{5(2x^2 + 1)}{2(x^2 + 9)}$ (2 marks)

(d) $x_1 = 0.3$
 $x_2 = 0.32453 \dots$ (1 mark)
 $x_3 = 0.33239 \dots$
 $= 0.3324$ 4dp (1 mark)

$$\frac{5(2 \text{Ans}^2 + 1)}{2(\text{Ans}^2 + 9)}$$

$x_4 = 0.33504 \dots$, $x_5 = 0.33595 \dots$
 $x_6 = 0.33626 \dots$, $x_7 = 0.33637 \dots$
 $x_8 = 0.33640 \dots$, $x_9 = 0.33641 \dots$
 $\left. \begin{matrix} x_7 & \& x_8 \end{matrix} \right\} x_7 \& x_8 \text{ agree to 4dp}$
 so $\alpha = 0.3364$ 4dp (1 mark)