7. Curve C has equation

$$y = (x^2 - 5x + 8)e^{x^2} \qquad x \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = (2x^3 - 10x^2 + 18x - 5)e^{x^2}$$

(3)

Given that

- C has only one stationary point
- the stationary point has x coordinate α
- $\frac{dy}{dx} \approx -0.5$ at $x = 0.3$
- $\frac{dy}{dx} \approx 0.9$ at $x = 0.4$

(b) explain why $0.3 < \alpha < 0.4$

(1)

(c) Show that α is a solution of the equation

$$x = \frac{5(2x^2 + 1)}{2(x^2 + 9)}$$

(3)

(d) Using the iteration formula

$$x_{n+1} = \frac{5(2x_n^2 + 1)}{2(x_n^2 + 9)} \qquad \text{with } x_1 = 0.3$$

find

(i) the value of x_3 to 4 decimal places,

(ii) the value of α to 4 decimal places.

(3)