

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1 - \cos 2\theta}{\sin^2 2\theta} \equiv k \sec^2 \theta \quad \theta \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

where k is a constant to be found. (b) $\frac{1 - \cos 2x}{\sin^2 2x} = \frac{1}{2} \sec^2 x$ from (a) (3)

$$\text{so, } \frac{1}{2} \sec^2 x = (1 + 2 \tan x)^2$$

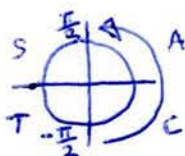
(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(b) contd

$$\tan x = -1 \Rightarrow x = -\frac{\pi}{4} \text{ (1 mark)}$$

$$\tan x = -\frac{1}{7} \Rightarrow x = -0.1418... \\ = 0.142 \text{ 3sf (1 mark)}$$

$$\frac{1 - \cos 2x}{\sin^2 2x} = (1 + 2 \tan x)^2$$



Give your answers to 3 significant figures where appropriate. (4)

(a) $RHS = k \sec^2 \theta = \frac{k}{\cos^2 \theta}$ ← try for only cos & sin for cancelling

$$LHS = \frac{1 - (2 \cos^2 \theta - 1)}{(2 \sin \theta \cos \theta)^2} \text{ (2 marks)}$$

← (use double angle formulae to convert '2θ' of LHS to 'θ' of RHS)

↙ (select this double angle formula for $\cos 2\theta$, because $\cos \theta$ on RHS)

$$= \frac{2 - 2 \cos^2 \theta}{4 \sin^2 \theta \cos^2 \theta} = \frac{2(1 - \cos^2 \theta)}{4 \sin^2 \theta \cos^2 \theta} = \frac{2 \sin^2 \theta}{4 \sin^2 \theta \cos^2 \theta}$$

(would have been quicker to use double angle formula $\cos 2\theta = 1 - 2 \sin^2 \theta$)

cancelling, $LHS = \frac{1}{2 \cos^2 \theta} = \frac{1}{2} \left(\frac{1}{\cos^2 \theta} \right) = \frac{1}{2} \sec^2 \theta = RHS$
with $k = \frac{1}{2}$ (1 mark)

(b) contd \sec^2 & \tan^2 are related by identity $\tan^2 + 1 = \sec^2$, so expand,

$$\frac{1}{2} \sec^2 x = 1 + 4 \tan x + 4 \tan^2 x$$

$$\sec^2 x = 2 + 8 \tan x + 8 \tan^2 x$$

$$\tan^2 x + 1 = 2 + 8 \tan x + 8 \tan^2 x \quad \left\{ \begin{array}{l} \text{we want quadratic in single function, which} \\ \text{we can solve} \end{array} \right.$$

$$\Rightarrow 7 \tan^2 x + 8 \tan x + 1 = 0$$

(1 mark)

solving quadratic, $\tan x = -1, -\frac{1}{7}$ (1 mark) ← (can use calculator)