

9. The vertical speed, $v \text{ m s}^{-1}$, of a skydiver, t seconds after their parachute opened, is modelled by the equation

$$v = A + Be^{-0.5t}$$

where A and B are constants.

Given that the vertical speed of the skydiver was

- 56 m s^{-1} at the instant the parachute opened
- 10 m s^{-1} exactly 5 seconds after the parachute opened

- (a) find a complete equation for the model.

Give the values of A and B to 3 significant figures.

(4)

Given also that the skydiver eventually descended safely to the ground at a constant vertical speed of 6 m s^{-1}

- (b) evaluate the model.

(2)

(a) Given $v = 56$ when $t = 0$, $56 = A + Be^{-0.5(0)}$
 $= A + Be^0 = A + B(1)$
 $= A + B$ (1 mark)

Given $v = 10$ when $t = 5$, $10 = A + Be^{-0.5(5)}$
 $= A + Be^{-2.5}$

$$\begin{array}{r} A + B = 56 \\ - (A + Be^{-2.5} = 10) \\ \hline B - Be^{-2.5} = 56 - 10 \end{array}$$

$$B(1 - e^{-2.5}) = 46$$

$$B = \frac{46}{1 - e^{-2.5}} = 50.11\dots = 50.1 \text{ 3sf (1 mark)}$$

$$\begin{aligned} A &= 56 - B = 56 - 50.11\dots = 5.886\dots \\ &= 5.89 \text{ 3sf (1 mark)} \end{aligned}$$

so, $v = 5.89 + 50.1e^{-0.5t}$ (1 mark)

(b) Model predicts that as $t \rightarrow \infty$, $e^{-0.5t} \rightarrow 0$
 $v \rightarrow A + B(0) = A$

so Model predicts 'constant' speed at limit of $A = 5.89 \text{ m s}^{-1}$ (1 mark)

5.89 m s^{-1} is close to observed "constant" speed of 6 m s^{-1}

so Model seems appropriate

(1 mark)