



10. (a) Sketch the graph with equation

$$y = |3x - 2a|$$

where  $a$  is a positive constant.

State the coordinates of each point where the graph cuts or meets the coordinate axes.

(2)

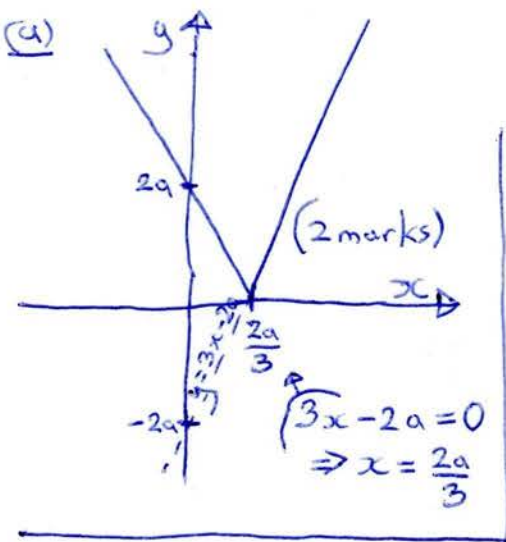
(b) Solve, in terms of  $a$ , the inequality

$$|3x - 2a| \leq x + a$$

(4)

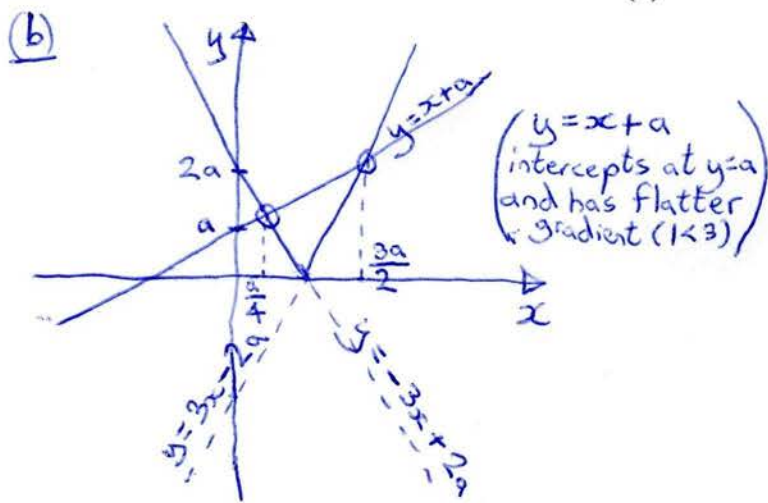
Given that  $|3x - 2a| \leq x + a$

(c) find, in terms of  $a$ , the range of possible values of  $g(x)$ , where



$$g(x) = 5a - \left| \frac{1}{2}a - x \right|$$

(3)



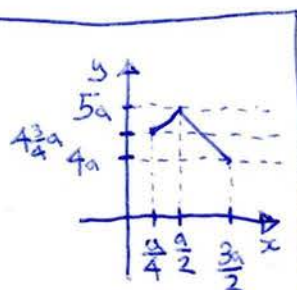
(c)  $\frac{a}{4} \leq x \leq \frac{3a}{2}$  from (b)

$x = \frac{a}{4} \Rightarrow g(x) = 5a - \left| \frac{1}{2}a - \frac{1}{4}a \right|$   
 $= 5a - \left| \frac{1}{4}a \right|$   
 $= 5a - \frac{1}{4}a$   
 $= 4\frac{3}{4}a$

$x = \frac{3a}{2} \Rightarrow g(x) = 5a - \left| \frac{1}{2}a - \frac{3}{2}a \right|$   
 $= 5a - |-a|$   
 $= 5a - a$   
 $= 4a$  (1 mark)

modulus  $\geq 0$  so maximum  $g(x) = 5a$  (1 mark)  
 (when  $\frac{1}{2}a - x = 0 \Rightarrow x = \frac{1}{2}a$ )

minimum value  $\leq g(x) \leq$  maximum value  $\Rightarrow 4a \leq g(x) \leq 5a$  (1 mark)



$3x - 2a = x + a$   
 $\Rightarrow x = \frac{3a}{2}$  (1 mark)

$-3x + 2a = x + a$   
 $\Rightarrow x = \frac{a}{4}$  (2 marks)

from sketch, we can see  $|3x - 2a|$  is below  $x + a$  where

$\frac{a}{4} \leq x \leq \frac{3a}{2}$  (1 mark)