

11. The mean yearly concentration, C parts per million (ppm), of carbon dioxide in the Earth's atmosphere was first measured in 1960.

The equation

$$C = ab^t \quad \text{where } a \text{ and } b \text{ are constants}$$

models the mean yearly concentration of carbon dioxide t years after 1960.

Given that the mean yearly concentration of carbon dioxide was

- 339 ppm in 1980
- 414 ppm in 2020

$$\left. \begin{array}{l} \text{(a)(i) Given } 339 = ab^{20} \\ \text{Given } 414 = ab^{60} \end{array} \right\} \frac{ab^{60}}{ab^{20}} = \frac{414}{339}$$

$$b^{40} = \frac{414}{339}$$

- (a) (i) find the value of b to 3 decimal places,
(ii) find the value of a to the nearest integer.

$$b = \left(\frac{414}{339}\right)^{\frac{1}{40}} \quad (1 \text{ mark}) \quad (4)$$

- (b) With reference to the model,

(a)(ii)

$$a = \frac{339}{b^{20}}$$

- (i) interpret the value of a ,

- (ii) interpret the value of b .

$$= \frac{339}{(1.0050\dots)^{20}} \quad (1 \text{ mark})$$

$$= 306.7\dots$$

$$= 307 \text{ to nearest int.} \quad (1 \text{ mark})$$

(2)

Using the model,

- (c) find the year when the mean yearly concentration of carbon dioxide is predicted to reach 450 ppm.

(Solutions based entirely on calculator technology are not acceptable.)

(4)

(b)(i) when $t=0$, $C = ab^0 = a(1) = a$

so a is the concentration of carbon dioxide in 1960 (1 mark)

(b)(ii) when $t=1$, $C = ab^1$

when $t=2$, $C = ab^2$

and so on, so b is the factor by which the concentration increases each year (1 mark)

(c) $450 = (306.7\dots)(1.0050\dots)^t$
 $(1.0050\dots)^t = \frac{450}{306.7\dots} \quad (1 \text{ mark})$

$$t = \log_{1.0050\dots} \left(\frac{450}{306.7\dots}\right) = 76.68\dots \quad (2 \text{ marks})$$

(c) contd 76 years after 1960 is 2036

77 years after 1960 is 2037 (1 mark)