

12.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\int x \sin kx \, dx = \frac{1}{k^2} \sin kx - \frac{1}{k} x \cos kx + c$$

where  $k$  is a constant and where  $c$  is an arbitrary constant.

(3)

A theme park ride lasts for 70 seconds.

The height above ground,  $H$  metres, of a passenger on the theme park ride is modelled by the differential equation

(b) Separating the Variables,  
 $10H \frac{dH}{dt} = 5t \sin\left(\frac{\pi t}{5}\right)$   
 (1 mark)

$$\frac{dH}{dt} = \frac{t \sin\left(\frac{\pi t}{5}\right)}{10H}$$

(b) contd  
 $5H^2 = \frac{1}{\left(\frac{\pi}{5}\right)^2} \sin\left(\frac{\pi t}{5}\right) - \frac{1}{\left(\frac{\pi}{5}\right)} t \cos\left(\frac{\pi t}{5}\right) + c$

$$0 \leq t \leq 70$$

where  $t$  seconds is the time from the start of the ride.

$$5H^2 = \frac{25}{\pi^2} \sin\left(\frac{\pi t}{5}\right) - \frac{5}{\pi} t \cos\left(\frac{\pi t}{5}\right) + c$$

(2 marks)

Given that the passenger is 5 m above ground at the start of the ride

(b) find the height above ground of the passenger 52 seconds after the start of the ride.

(6)

(a)

$$\int x \sin kx \, dx$$

$u = x$   
 $u' = 1$   
 $v' = \sin kx$   
 $v = -\frac{1}{k} \cos kx$

Integration by Parts:

$$\int u v' = uv - \int u' v$$

to reduce complexity of integral,  
 select power of  $x$  as  $u$ , or  $\ln x$  as  $u$   
 if  $\ln x$  is present

$$uv - \int u' v = x \left(-\frac{1}{k} \cos kx\right) - \int (1) \left(-\frac{1}{k}\right) \cos kx \quad (1 \text{ mark})$$

$$= -\frac{1}{k} x \cos kx + \frac{1}{k} \int \cos kx \quad (1 \text{ mark})$$

$$= -\frac{1}{k} x \cos kx + \frac{1}{k} \left(\frac{1}{k} \sin kx\right) + c$$

$$= \frac{1}{k^2} \sin kx - \frac{1}{k} x \cos kx + c \quad (1 \text{ mark})$$

(b) contd Given  $H=5$  when  $t=0$ ,

$$5(5)^2 = \frac{25}{\pi^2} \sin(0) - \frac{5}{\pi} (0) \cos(0) + c$$

$$125 = 0 - 0 + c$$

$$\Rightarrow c = 125 \quad (1 \text{ mark})$$

(b) contd When  $t=52$ ,

$$5H^2 = \frac{25}{\pi^2} \sin\left(\frac{52\pi}{5}\right) - \frac{5}{\pi} (52) \cos\left(\frac{52\pi}{5}\right) + 125$$

$$H = \sqrt{\frac{5}{\pi^2} \sin\left(\frac{52\pi}{5}\right) - \frac{52}{\pi} \cos\left(\frac{52\pi}{5}\right) + 25} = 4.512 \dots = 4.51 \text{ m}$$

3sf (2 marks)