

13. Given that

$$y^2 - x^2 = 8$$

show that

$$\frac{d^2y}{dx^2} = \frac{8}{y^3}$$

(5)

Implicit Differentiation:

$$\frac{d(y^2)}{dx} - \frac{d(x^2)}{dx} = \frac{d(8)}{dx}$$

$$2y \frac{dy}{dx} - 2x = 0 \quad (1 \text{ mark})$$

by Chain Rule

$$\frac{dy}{dx} = \frac{x}{y} \quad \text{Now Quotient Rule for } \frac{d^2y}{dx^2}$$

$$u = x \quad \frac{du}{dx} = 1$$

$$v = y \quad \frac{dv}{dx} = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{u'v - uv'}{v^2}$$

$$= \frac{(1)y - x \frac{dy}{dx}}{y^2} = \frac{y - x \frac{dy}{dx}}{y^2} \quad (2 \text{ marks})$$

$$\text{But, } \frac{dy}{dx} = \frac{x}{y}, \text{ so } \frac{d^2y}{dx^2} = \frac{y - x \left(\frac{x}{y}\right)}{y^2} = \frac{\frac{y^2 - x^2}{y}}{y^2} = \frac{y^2 - x^2}{y^3} \quad (1 \text{ mark})$$

$$\text{But, } y^2 - x^2 = 8, \text{ so } \frac{d^2y}{dx^2} = \frac{8}{y^3} \quad (1 \text{ mark})$$