



(3)

(4)

14. (i) Prove that the sum of the squares of 2 consecutive odd integers is always 2 more than a multiple of 8

(ii) Use proof by contradiction to show that $\log_2 5$ is irrational.

(i) Let the 2 consecutive odd integers be $(2n-1)$ and $(2n+1)$

$$(2n-1)^2 + (2n+1)^2 = 4n^2 - 4n + 1 + 4n^2 + 4n + 1 \quad (1 \text{ mark})$$

$$= 8n^2 + 2 \quad (1 \text{ mark})$$

so, the sum of squares of 2 consecutive odd integers is always 2 more than a multiple of 8. (1 mark)

(ii) Assume $\log_2 5$ is rational

then $\log_2 5 = \frac{a}{b}$ where a and b are integers (1 mark)

then by definition $5 = 2^{\frac{a}{b}}$ (1 mark)

$$\Rightarrow 5^b = 2^a \quad (1 \text{ mark})$$

but, an integer power of 2 cannot equal an integer power of 5 because they are both primes without a common factor.

So, because of this resulting contradiction,

$\log_2 5$ must be irrational (1 mark)