Question	Scheme	Marks	AOs
14(a)	Attempts to use both $\sin(x-60°)=\pm\sin x\cos 60°\pm\cos x\sin 60°$ $\cos(x-30°)=\pm\cos x\cos 30°\pm\sin x\sin 30°$	M1	2.1
	Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$	A1	1.1b
	Either uses $\frac{\sin x}{\cos x}=\tan x$ and attempts to make $\tan x$ the subject E.g. $(2\cos 60°-\sin 30°)\tan x=\cos 30°+2\sin 60°$ Or attempts $\sin 30°$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ E.g. $\left(2\times\frac{1}{2}-\frac{1}{2}\right)\sin x=\left(2\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\cos x$,	M1	2.1
	Proceeds to given answer showing all key steps E.g. $\frac{1}{2}\tan x=\frac{3\sqrt{3}}{2}\Rightarrow\tan x=3\sqrt{3}$ *	A1*	1.1b
		(4)	
(b)	Deduces that $x=2\theta+60°$	B1	2.2a
	$\tan(2\theta+60°)=3\sqrt{3}\Rightarrow 2\theta+60°=79.1°,259.1°,.....$	M1	1.1b
	Correct method to find one value of θ E.g $\theta=\frac{79.1°-60°}{2}$	dM1	1.1b
	$\theta=$ awrt $9.6°, 99.6°$ (See note)	A1	2.1
		(4)	
		(8 marks)	
Notes:			

(a)

M1: Attempts to use both compound angle expansions to set up an equation in sin x and cos x
The terms must be correct but condone sign errors and a slip on the multiplication of 2

A1: Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$ o.e.
Note that $\cos 60°=\sin 30°$ and $\cos 30°=\sin 60°$
Also allow this mark for candidates who substitute in their trigonometric values "early"

$$2\sin x\times\frac{1}{2}-2\cos x\times\frac{\sqrt{3}}{2}=\cos x\times\frac{\sqrt{3}}{2}+\sin x\times\frac{1}{2}\quad \text{o.e.}$$

M1: Shows the necessary progress towards showing the given result.
There are three key moves, two of which must be shown for this mark.

- uses $\frac{\sin x}{\cos x}=\tan x$ to form an equation in just tan x.
- uses exact numerical values for $\sin 30°, \sin 60°, \cos 30°, \cos 60°$ with at least two correct
- collects terms in $\sin x$ and $\cos x$ or alternatively in $\tan x$

A1*: Proceeds to the given answer with accurate work showing all necessary lines.

Examples of two proofs showing all necessary lines

E.g. I $\quad 2\sin x\cos 60° - 2\cos x\sin 60° = \cos x\cos 30° + \sin x\sin 30°$

$\sin x\left(2\cos 60° - \sin 30°\right) = \cos x\left(\cos 30° + 2\sin 60°\right)$ 1. collect terms

$\left(2\cos 60° - \sin 30°\right)\tan x = \cos 30° + 2\sin 60°$ 2. $\dfrac{\sin x}{\cos x} = \tan x$ so M1

$\tan x = \dfrac{\cos 30° + 2\sin 60°}{2\cos 60° - \sin 30°} = \dfrac{\frac{\sqrt{3}}{2} + \sqrt{3}}{1 - \frac{1}{2}} = 3\sqrt{3}$ 3..uses values and completes proof A1*

E.g II

$2\sin x \times \frac{1}{2} - 2\cos x \times \frac{\sqrt{3}}{2} = \cos x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{1}{2}$ 1.uses values

$\Rightarrow \frac{1}{2}\sin x = \frac{3\sqrt{3}}{2}\cos x$ 2.collects terms so M1

$\Rightarrow \tan x = 3\sqrt{3}$

3. $\dfrac{\sin x}{\cos x} = \tan x$ completes proof A1*

(b) Hence

B1: Deduces that $x = 2\theta + 60°$ o.e such as $\theta = \dfrac{x - 60°}{2}$

This is implied for sight of the equation $\tan\left(2\theta + 60°\right) = 3\sqrt{3}$

M1: Proceeds from $\tan\left(2\theta \pm \alpha°\right) = 3\sqrt{3} \Rightarrow 2\theta \pm \alpha° =$ one of $79.1°, 259.1°, ...$ where $\alpha \neq 0$

One angle for $\arctan\left(3\sqrt{3}\right)$ **must** be correct in degrees or radians(3sf). FYI radian answers 1.38, 4.52

dM1: Correct method to find one value of θ from their $2\theta \pm \alpha° = 79.1°$ to $\theta = \dfrac{79.1° \mp \alpha°}{2}$

This is dependent upon one angle being correct, which must be in degrees, for $\arctan\left(3\sqrt{3}\right)$

$\tan\left(2\theta + 60°\right) = 3\sqrt{3} \Rightarrow \theta = 9.6°$ would imply B1 M1 dM1

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range

Otherwise: Via the use of $\cos\left(2\theta + 30°\right) = \cos 2\theta\cos 30° - \sin 2\theta\sin 30°$.

$2\sin 2\theta = \cos\left(2\theta + 30°\right) \Rightarrow \tan 2\theta = \dfrac{\sqrt{3}}{5} \Rightarrow \theta = 9.6°, 99.6°$

The order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta = \dfrac{\sqrt{3}}{5}$ o.e so allow $\tan 2\theta =$ awrt 0.346 or $\tan 2\theta = \dfrac{\cos 30°}{2 + \sin 30°}$

Or via double angle identities $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities to reach a form $\tan 2\theta = k$ where k is a constant not $3\sqrt{3}$ (or expression in trig terms such as cos 30 etc as seen above)

Or via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta = k$ leading to $\theta = ...$

Correctly solves their $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ leading to $\theta = ...$

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range.

Note that $\tan\left(2\theta + 60°\right) = 3\sqrt{3} \Rightarrow \theta = 9.6°, 99.6°$ is acceptable for full marks