

4. (a) Express  $\lim_{\delta x \rightarrow 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$  as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \rightarrow 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

where  $k$  is a constant to be found.

(2)

(a)  $\int_{2.1}^{6.3} \frac{2}{x} dx$  (1 mark)

(b)  $= [2 \ln x]_{2.1}^{6.3}$

$$= (2 \ln 6.3) - (2 \ln 2.1) \quad (1 \text{ mark})$$

$$= 2 (\ln 6.3 - \ln 2.1)$$

$$= 2 \ln \left( \frac{6.3}{2.1} \right) = 2 \ln 3 = \ln 3^2 = \ln 9$$

(1 mark)