

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x+y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

(i) Assume there exist two integers p, q such that pq is even and p, q are both odd (1 mark)

$$\begin{aligned} p \text{ odd} &\Rightarrow p = 2m + 1 && \text{for some integer } m \\ q \text{ odd} &\Rightarrow q = 2n + 1 && \text{for some integer } n \\ pq &= (2m + 1)(2n + 1) && (1 \text{ mark}) \end{aligned}$$

$$= 4mn + 2m + 2n + 1$$

$$= 2(2mn + m + n) + 1$$

$$= \text{odd number} \Rightarrow \text{a contradiction}$$

so, if pq is even p, q cannot both be odd

\Rightarrow at least one of p, q must be even (1 mark)

$$\begin{aligned} \text{(ii)} \quad (x+y)^2 &< 9x^2 + y^2 \\ \Rightarrow x^2 + 2xy + y^2 &< 9x^2 + y^2 \\ \Rightarrow 2xy &< 8x^2 && (1 \text{ mark}) \end{aligned}$$

as $x < 0$, when we divide both sides of the inequality by x , the inequality reverses, so

$$\begin{aligned} 2y &> 8x \\ \div 2 & \quad \div 2 \\ y &> 4x && (1 \text{ mark}) \end{aligned}$$