

8.

(b) continued (1 mark)

$$= \frac{10 - 0.4t - (0.4t + 0.4)\ln(t+1)}{t+1}$$

$$\frac{dv}{dt} = 0 \text{ when numerator} = 0$$

$$\Rightarrow 10 - 0.4t - (0.4t + 0.4)\ln(t+1) = 0$$

$$\Rightarrow t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)}$$

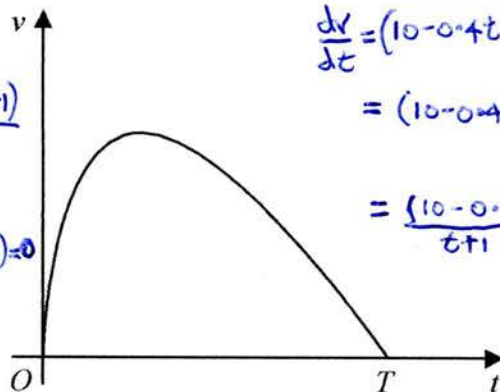


Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v \text{ ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t) \ln(t + 1) \quad 0 \leq t \leq T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model, (a) $v = 0$ when $\ln(t+1) = 0 \Rightarrow t+1 = 1 \Rightarrow t = 0$

$$\text{and when } (10 - 0.4t) = 0 \Rightarrow t = \frac{10}{0.4} = 25 = T$$

(a) find the value of T

(1 mark)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

$$(c)(i) \quad t_2 = \frac{26}{1 + \ln(7+1)} - 1 = 7.443... \quad (1 \text{ mark})$$

$$t_3 = \frac{26}{1 + \ln(7.443...)} - 1 = 7.2978... \\ = 7.298 \text{ 3dp} \quad (1 \text{ mark})$$

(b) continued

$$t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)} \quad (1 \text{ mark})$$

$$= \frac{25 + 1 - 1 - \ln(t+1)}{1 + \ln(t+1)}$$

$$= \frac{26 - (1 + \ln(t+1))}{1 + \ln(t+1)} = \frac{26}{1 + \ln(t+1)} - 1$$

$$t_4 = 7.3440... \quad t_9 = 7.3327...$$

$$t_5 = 7.3292... \quad t_8 \text{ \& } t_9 \text{ agree to 3dp}$$

$$t_6 = 7.3339... \Rightarrow t = 7.3335 \text{ 3dp}$$

$$t_7 = 7.3324... \quad t_8 = 7.3329... \quad (1 \text{ mark})$$