

Figure 3

Figure 3 shows a sketch of a parallelogram $PQRS$.

Given that

- $\vec{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\vec{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram $PQRS$ is a rhombus.

(2)

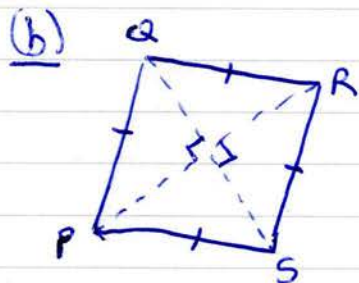
(b) Find the exact area of the rhombus $PQRS$.

(4)

$$\text{(a) length } |\vec{PQ}| = \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{29}$$

$$\text{length } |\vec{QR}| = \sqrt{5^2 + (-2)^2} = \sqrt{29} \quad (1 \text{ mark})$$

parallelogram with equal side lengths is rhombus
so $|\vec{PQ}| = |\vec{QR}| \Rightarrow$ rhombus (1 mark)



$$\begin{aligned} \text{Area } PQRS &= 2 \times \text{Area } \triangle PQR, \text{ by symmetry} \\ &= 2 \times \left(\frac{1}{2} \times b \times h\right) \\ &= 2 \times \frac{1}{2} \times |\vec{PR}| \times \frac{1}{2} |\vec{QS}| \\ &= \frac{1}{2} \times |\vec{PR}| \times |\vec{QS}| \quad (1 \text{ mark}) \end{aligned}$$

$$\vec{PR} = \vec{PQ} + \vec{QR} = (2+5)\mathbf{i} + (3+0)\mathbf{j} + (-4-2)\mathbf{k} = 7\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$$

$$\vec{QS} = \vec{QP} + \vec{PS} = -\vec{PQ} + \vec{QR} = (-2+5)\mathbf{i} + (-3+0)\mathbf{j} + (4-2)\mathbf{k} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} \quad (2 \text{ marks})$$

$$\frac{1}{2} \times |\vec{PR}| \times |\vec{QS}| = \frac{1}{2} \sqrt{7^2 + 3^2 + (-6)^2} \sqrt{3^2 + (-3)^2 + 2^2} = \frac{1}{2} \sqrt{94 \times 22} = \sqrt{517} \quad (1 \text{ mark})$$