

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands,  $N_b$ , is modelled by the equation

$$N_b = 45 + 220e^{0.05t}$$

where  $t$  is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study,

(a) when  $t=0$ ,  

$$N_b = 45 + 220e^0$$

$$= 45 + 220$$

$$= 265 \text{ thousand (1 mark)}$$
 (1)

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year.

(3)

The number of wasps, measured in thousands,  $N_w$ , is modelled by the equation

$$N_w = 10 + 800e^{-0.05t}$$

where  $t$  is the number of years from the start of the study.

When  $t = T$ , according to the models, there are an equal number of bees and wasps.

(c) Find the value of  $T$  to 2 decimal places.

(4)

(b)  $\frac{dN_b}{dt} = 0 + (220e^{0.05t} \times 0.05) = 11e^{0.05t}$  (1 mark)

when  $t=10$ ,  $\frac{dN_b}{dt} = 11e^{0.05(10)}$  (1 mark)

$$= 18.1359... \Rightarrow 18,000 \text{ to nearest '000}$$
 (1 mark)

(c)  $N_b = N_w \Rightarrow 45 + 220e^{0.05t} = 10 + 800e^{-0.05t}$

$$\Rightarrow 220e^{0.05t} + 35 - 800e^{-0.05t} = 0$$
 (1 mark)

$$\Rightarrow 220(e^{0.05t})^2 + 35(e^{0.05t}) - 800 = 0$$
 (1 mark)

$$\Rightarrow 44(e^{0.05t})^2 + 7(e^{0.05t}) - 160 = 0$$

$$\Rightarrow e^{0.05t} = 1.829... ; -1.988...$$

negative exponential not possible

so  $e^{0.05t} = 1.829...$  (1 mark)

$$\Rightarrow 0.05t = \ln(1.829...)$$

$$\Rightarrow T = \frac{\ln(1.829...)}{0.05} = 12.075... = 12.08 \text{ years 2dp}$$
 (1 mark)