

12.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

$$\int_1^{e^2} x^3 \ln x \, dx$$

$$\begin{array}{l} \frac{dv}{dx} \\ \uparrow \\ v = \frac{1}{4}x^4 \end{array} \quad \begin{array}{l} u \\ \uparrow \\ \frac{du}{dx} = \frac{1}{x} \end{array}$$

$$= uv - \int v \frac{du}{dx}$$

$$= \left[\ln x \left(\frac{1}{4} x^4 \right) \right]_1^{e^2} - \int_1^{e^2} \frac{1}{4} x^4 \left(\frac{1}{x} \right) dx \quad (1 \text{ mark})$$

$$= \left[\frac{x^4}{4} \ln x \right]_1^{e^2} - \int_1^{e^2} \frac{x^3}{4} dx$$

$$= \left[\frac{x^4}{4} \ln x \right]_1^{e^2} - \left[\frac{x^4}{16} \right]_1^{e^2} \quad (2 \text{ marks})$$

$$= \left(\frac{(e^2)^4}{4} \ln(e^2) - \frac{1^4}{4} \ln(1) \right) - \left(\frac{(e^2)^4}{16} - \frac{1^4}{16} \right) \quad (1 \text{ mark})$$

$$= \frac{e^8}{4} (2) - \frac{1}{4} (0) - \frac{e^8}{16} + \frac{1}{16}$$

$$= \frac{7}{16} e^8 + \frac{1}{16} \quad (1 \text{ mark})$$