13. (i) In an arithmetic series, the first term is a and the common difference is d. Show that

S_n =
$$\frac{n}{2} \left[2a + (n-1)d \right]$$

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the

weekly amounts he saves form an arithmetic sequence. Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$
(a) Hence state the number of weeks larger taken to be

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer. (1)

 $\frac{(i)}{5} = a + (a+d) + (a+2d)$ + ... + (a+(n-2)d) + (a+(n-1)d)

$$\frac{(5)}{5} = a + (a+d) + (a+2d) + ... + (a+(n-2)d) + (a+(n-1)d)$$

$$+ \frac{5}{n} = (a+(n-1)d) + (a+(n-2)d) + ... + (a+d) + a + a + a + b$$

$$25_n = 2a+(n-1)d + 2a+(n-1)d + ... + (2a+(n-1)d) + (2a+(n-1)d)$$

$$= n \times (2a + (n-1)d) \implies S_n = \frac{n}{2}(2a + (n-1)d) (2 marks)$$

$$\frac{(ii)(a)}{(a)} 64 = \frac{n}{2} (2(10) + (n-1)(-0.8))$$

$$128 = 20n - 0.8n^{2} + 0.8n$$

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$$0.8n^{2} - 20.8n + 128 = 0$$

$$0.8n^{2} - 20n = 0.00 + 0.00$$

$$0.8n^{2} - 20.8n + 12.8 = 0$$

$$0.8 \Rightarrow n^{2} - 26n + 160 = 0$$

$$128 = 20n - 0.8n^{2} + 0.00$$

$$0.8n^{2} - 20.8n + 129 = 0$$

$$128 = 20n - 0.8n^{2} + 129 = 0$$

$$128 = 20n - 0.8n^{2} + 100 = 0$$

$$18n^2 - 20.8n + 128 = 0$$

 $18n^2 - 26n + 160 = 0$

$$0.8n^{2} - 20.8n + 128 = 0$$

 $10.8 \Rightarrow n^{2} - 26n + 160 = 0$
(11)(b) $(n-10)(n-16) = 0 \Rightarrow n = 10,16$

$$0.8n^2 - 20.8n + 128 = 0$$

 $10.8 \Rightarrow n^2 - 26n + 160 = 0$

$$128 = 20n - 0.8n^{2} + 0$$

$$0.8n^{2} - 20.8n + 128 = 0$$

$$128 = 20n - 0.8n^{2} + 160 = 0$$

$$128 = 20n - 0.8n^{2} + 160 = 0$$

$$128 = 20n - 0.8n^{2} + 0.00$$

$$0.8n^{2} - 20.8n + 128 = 0$$

(iik) after 10 weeks, James has reached his goal (Imark) between 10 and 16 weeks, James would start saving negative amounts

(1 mark)

(Imark)

(3)

(2)

(1)