

13. (i) In an arithmetic series, the first term is a and the common difference is d .

Show that

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad (3)$$

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0 \quad (2)$$

(b) Solve the equation

$$n^2 - 26n + 160 = 0 \quad (1)$$

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

$$(i) S_n = a + (a+d) + (a+2d) + \dots + (a+(n-2)d) + (a+(n-1)d)$$

$$+ S_n = (a+(n-1)d) + (a+(n-2)d) + \dots + (a+d) + a \quad (1 \text{ mark})$$

$$2S_n = 2a + (n-1)d + 2a + (n-1)d + \dots + (2a + (n-1)d) + (2a + (n-1)d)$$

$$= n \times (2a + (n-1)d) \Rightarrow S_n = \frac{n}{2} (2a + (n-1)d) \quad (2 \text{ marks})$$

$$(ii)(a) 64 = \frac{n}{2} (2(10) + (n-1)(-0.8)) \quad (1 \text{ mark})$$

$$128 = 20n - 0.8n^2 + 0.8n$$

$$0.8n^2 - 20.8n + 128 = 0$$

$$\div 0.8 \Rightarrow n^2 - 26n + 160 = 0 \quad (1 \text{ mark})$$

$$(ii)(b) (n-10)(n-16) = 0 \Rightarrow n = 10, 16 \quad (1 \text{ mark})$$

(ii)(c) after 10 weeks, James has reached his goal (1 mark)
between 10 and 16 weeks, James would start saving negative amounts