

14.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2 \sin(x - 60^\circ) = \cos(x - 30^\circ)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leq \theta < 180^\circ$

$$2 \sin 2\theta = \cos(2\theta + 30^\circ)$$

giving your answers to one decimal place.

(4)

(a) $2 \sin(x - 60^\circ) = \cos(x - 30^\circ)$ (2 marks)
 using Addition Formulae,
 $2(\sin x \cos 60^\circ - \cos x \sin 60^\circ) = \cos x \cos 30^\circ + \sin x \sin 30^\circ$

$$\Rightarrow 2\left(\sin x \left(\frac{1}{2}\right) - \cos x \left(\frac{\sqrt{3}}{2}\right)\right) = \cos x \left(\frac{\sqrt{3}}{2}\right) + \sin x \left(\frac{1}{2}\right)$$

$$\sin x - \sqrt{3} \cos x = \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x$$

$$\sin x - \frac{1}{2} \sin x = \frac{\sqrt{3}}{2} \cos x + \sqrt{3} \cos x$$

$$\frac{1}{2} \sin x = \frac{3\sqrt{3}}{2} \cos x$$

$$\sin x = 3\sqrt{3} \cos x$$

$$\frac{\sin x}{\cos x} = 3\sqrt{3} = \tan x \quad (2 \text{ marks})$$

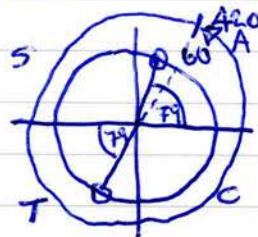
(b) if $2\theta = x - 60^\circ$
 then $x = 2\theta + 60^\circ$
 and $x - 30^\circ = 2\theta + 60^\circ - 30^\circ$
 $= 2\theta + 30^\circ$
 (1 mark)

so,
 $\tan(2\theta + 60^\circ) = 3\sqrt{3}$
 $\Rightarrow 2\theta + 60^\circ = \tan^{-1}(3\sqrt{3})$
 $= 79.106\dots$
 (1 mark)

$$0 \leq \theta < 180$$

$$0 \leq 2\theta < 360$$

$$60 \leq 2\theta + 60 < 420$$



$$2\theta + 60^\circ = 79.106\dots \quad 180 + 79.106\dots$$

$$\theta = \frac{79.106\dots - 60}{2}, \quad \frac{259.106\dots - 60}{2}$$

$$= 9.6^\circ; \quad 99.6^\circ \text{ 1dp (2 marks)}$$