

15.

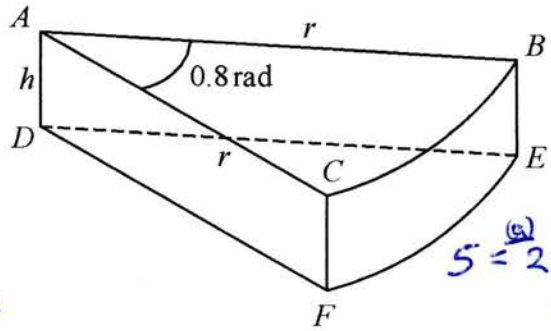


Figure 5

(a) Area Sector ABC  
 $= \frac{1}{2} r^2 (0.8) = 0.4r^2$   
 Area Curved Face BCFE  
 $= \text{arc length } BC \times h$   
 $= r(0.8)h = 0.8rh$

$S = 2 \times \text{Sector Area} + 2 \times \text{rectangle area} + \text{Curved Face Area}$   
 $= 2(0.4r^2) + 2(rh) + 0.8rh$   
 $= 0.8r^2 + 2.8rh$   
 (1 mark)

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius  $r$  cm and centre A
- angle  $BAC = 0.8$  radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length  $h$  cm

Given that the volume of the toy is  $240 \text{ cm}^3$

(a) show that the surface area of the toy,  $S \text{ cm}^2$ , is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(a) Volume = Sector Area  $\times h$   
 $240 = 0.4r^2 \times h$   
 required equation has no  $h$ ,  
 so we need to eliminate  $h$   
 using Volume equation,  
 $h = \frac{240}{0.4r^2}$  (2 marks)

Subst. for  $h$ ,  
 $S = 0.8r^2 + 2.8r \left( \frac{240}{0.4r^2} \right)$  (1 mark)  
 $= 0.8r^2 + \frac{1680}{r}$

Using algebraic differentiation,

(b) find the value of  $r$  for which  $S$  has a stationary point.

(c) Prove, by further differentiation, that this value of  $r$  gives the minimum surface area of the toy.

(b)  $\frac{dS}{dr} = \frac{d(0.8r^2 + 1680r^{-1})}{dr} = 1.6r - 1680r^{-2} = 1.6r - \frac{1680}{r^2}$  (2 marks)

$\frac{dS}{dr} = 0 \Rightarrow 1.6r - \frac{1680}{r^2} = 0 \Rightarrow 1.6r = \frac{1680}{r^2} \Rightarrow r^3 = 1050$   
 $\Rightarrow r = \sqrt[3]{1050} = 10.163\dots$  (2 marks)

(c)  $\frac{d^2S}{dr^2} = 1.6 + 2(1680)r^{-3}$   
 when  $r = 10.163\dots$   $\frac{d^2S}{dr^2} = 4.8$  (1 mark)  
 $\frac{d^2S}{dr^2} > 0$  at  $r = 10.163\dots$   
 hence  $S$  is minimum at  $r = 10.163\dots$  (1 mark)