

16.

(a) contd.  
 $= \int 16 \sin^2 2t + 24 \sin t \sin 2t dt$  (1 mark)  
 $\cos 2\theta = 1 - 2\sin^2 \theta$ ,  
 so  $16 \sin^2 2t$   
 $= (1 - \cos 4t) 8$   
 $= 8 - 8 \cos 4t$  (1 mark)

(a)  $\frac{dx}{dt} = 16 \sin t \cos t = 8(2 \sin t \cos t) = 8 \sin 2t$   
 $\Rightarrow dx = 8 \sin 2t dt$   
 $\int y dx = \int (2 \sin 2t + 3 \sin t)(8 \sin 2t) dt$  (1 mark)

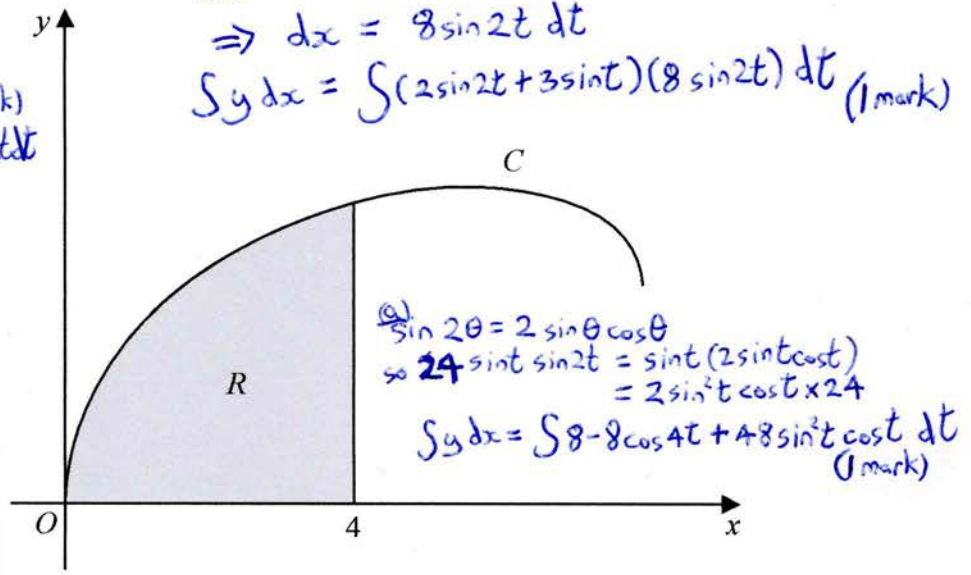


Figure 6

Figure 6 shows a sketch of the curve  $C$  with parametric equations

$$x = 8 \sin^2 t \quad y = 2 \sin 2t + 3 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region  $R$ , shown shaded in Figure 6, is bounded by  $C$ , the  $x$ -axis and the line with equation  $x = 4$

(a) Show that the area of  $R$  is given by

$$\int_0^a (8 - 8 \cos 4t + 48 \sin^2 t \cos t) dt$$

where  $a$  is a constant to be found.

(a) contd.  $x = 4 \mid t = \sin^{-1} \sqrt{\frac{x}{8}}$   
 $0 \mid 0$   
 $4 \mid \sin^{-1} \sqrt{\frac{4}{8}} = \frac{\pi}{4}$   
 so  $\int_{x=0}^{x=4} = \int_{t=0}^{t=\frac{\pi}{4}} \quad a = \frac{\pi}{4}$  (1 mark)

(5)

(b) Hence, using algebraic integration, find the exact area of  $R$ .

(4)

(b)  $\left[ 8t - \left(\frac{8}{4}\right) \sin 4t + \frac{48}{3} \sin^3 t \right]_0^{\frac{\pi}{4}}$   
 $= \left[ 8t - 2 \sin 4t + 16 \sin^3 t \right]_0^{\frac{\pi}{4}}$  (1 mark)  
 $= \left( \frac{8\pi}{4} - 2 \sin \pi + 16 \left( \sin \frac{\pi}{4} \right)^3 \right) - \left( 0 - 2 \sin 0 + 16 \left( \sin 0 \right)^3 \right)$   
 $= 2\pi - 2(0) + 16 \left( \frac{1}{\sqrt{2}} \right)^3 - 0 + 2(0) + 16(0)^3$   
 $= 2\pi + 2^{4-\frac{3}{2}} + 2^4 \left( \frac{1}{2^{\frac{3}{2}}} \right)$   
 $= 2\pi + 2^{\frac{5}{2}} = 2\pi + 2^2 2^{\frac{1}{2}} = 2\pi + 4\sqrt{2}$  (2 marks)