

3. The functions f and g are defined by

$$f(x) = e^{3x} + 2 \quad x \in \mathbb{R}$$

$$g(x) = \ln x \quad x \in \mathbb{R} \quad x > 0$$

(a) State the range of f

(1)

(b) Find $fg(x)$, giving your answer in simplest form.

(2)

(c) Find f^{-1}

(3)

$$\begin{aligned} \underline{\text{(a)}} \quad e^x > 0 &\Rightarrow e^{3x} > 0 \\ &\Rightarrow e^{3x} + 2 > 2 \\ &\Rightarrow f(x) > 2 \end{aligned} \quad (1 \text{ mark})$$

$$\begin{aligned} \underline{\text{(b)}} \quad fg(x) &= f(g(x)) \\ &= e^{3g(x)} + 2 \\ &= e^{3 \ln x} + 2 \\ &= e^{\ln x^3} + 2 \quad (1 \text{ mark}) \\ &= x^3 + 2 \quad (1 \text{ mark}) \end{aligned}$$

$$\underline{\text{(c)}} \quad y = e^{3x} + 2$$

$$\begin{aligned} \text{make } x \text{ the subject } &\Rightarrow e^{3x} = y - 2 \\ 3x &= \ln(y - 2) \quad (1 \text{ mark}) \\ x &= \frac{1}{3} \ln(y - 2) \end{aligned}$$

$$\text{so } f^{-1}(x) = \frac{1}{3} \ln(x - 2) \quad (1 \text{ mark})$$

Domain of f^{-1} is Range of f

$$f^{-1}(x) = \frac{1}{3} \ln(x - 2), \quad x > 2 \quad (1 \text{ mark})$$