

6. Prove by contradiction that for all positive real values of x

$$x + \frac{4}{x} \geq 4$$

Assume conjecture is not true

Then $x + \frac{4}{x} < 4$ for some Real x (1 mark)

Because x is positive, if we multiply both sides of the inequality by x , it will not reverse the inequality:

$$\Rightarrow x^2 + 4 < 4x$$

$$x^2 - 4x + 4 < 0 \quad (1 \text{ mark})$$

$$(x-2)^2 < 0 \quad (1 \text{ mark})$$

but this is a contradiction, because $(x-2)^2 \geq 0$ for all Real x

Hence, assuming conjecture is false leads to a contradiction,
Hence, conjecture must be true:

$$x + \frac{4}{x} \geq 4 \quad (1 \text{ mark})$$