

8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

FINEVIEW

(a) Given that $\cos 2\theta + \cos \theta + 1 \neq 0$, show that

$$\frac{\sin 2\theta + \sin \theta}{\cos 2\theta + \cos \theta + 1} \equiv \tan \theta$$

(3)

(b) Hence solve, for $0 < x < \pi$

$$\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = 2 \cos x$$

Give your answers to 3 decimal places.

(5)

(a) to get from LHS to RHS

we need to use double-angle formulae to reduce 2θ to θ :

$$\text{LHS} = \frac{2 \sin \theta \cos \theta + \sin \theta}{\cos 2\theta + \cos \theta + 1} = \frac{\sin \theta (2 \cos \theta + 1)}{\cos 2\theta + \cos \theta + 1} \quad (1 \text{ mark})$$

there is a choice of double-angle formulae for $\cos 2\theta$.Looking ahead, maybe " $\cos 2\theta = 2\cos^2 \theta - 1$ " will allow cancelling:

$$= \frac{\sin \theta (2 \cos \theta + 1) (1 \text{ mark})}{2 \cos^2 \theta - 1 + \cos \theta + 1} = \frac{\sin \theta (2 \cos \theta + 1)}{2 \cos^2 \theta + \cos \theta} = \frac{\sin \theta (2 \cos \theta + 1)}{\cos \theta (2 \cos \theta + 1)}$$

$$= \frac{\sin \theta}{\cos \theta} = \tan \theta = \text{RHS} \quad (1 \text{ mark})$$

$$(b) \text{ from (a), } \frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \frac{\sin x}{\cos x} = 2 \cos x$$

$$\Rightarrow \sin x = 2 \cos^2 x \Rightarrow 2 \cos^2 x - \sin x = 0$$

$$\Rightarrow 2(1 - \sin^2 x) - \sin x = 0 \Rightarrow -2 \sin^2 x - \sin x + 2 = 0$$

$$\Rightarrow 2 \sin^2 x + \sin x - 2 = 0 \quad (2 \text{ marks})$$

$$\text{From Calculator or Quadratic Formula, } \sin x = \frac{-1 \pm \sqrt{17}}{4} \quad (1 \text{ mark}) \quad \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$x = \sin^{-1} \left(\frac{-1 + \sqrt{17}}{4} \right), \sin^{-1} \left(\frac{-1 - \sqrt{17}}{4} \right) = 0.8959\dots, 2.2456\dots$$

$$0.7807\dots$$

$$\frac{-1 - \sqrt{17}}{4} < -1$$

so \sin^{-1} not possible



$$\pi - 0.8959\dots$$

$$= 0.896, 2.246 \text{ 3dp} \quad (2 \text{ marks})$$