

9. A kettle contains a quantity of water.

The water is cooling down.

The temperature, $\theta^\circ\text{C}$, of the water at time t minutes after cooling began is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are constants.

(a) when $t=0$, $\theta=80$, so $80 = 21 + Ae^{(0)} = 21 + A$
 $\Rightarrow A = 80 - 21 = 59$ (1 mark)

Given that

when $t=40$, $\theta=33$, so $33 = 21 + 59e^{-k(40)}$

- at the instant cooling began, the temperature of the water was 80°C
- 40 minutes after cooling began, the temperature of the water was 33°C

(a) find a complete equation for the model. Give the exact value of A and the value of k to 3 significant figures.

(a) contd so $e^{-40k} = \frac{33-21}{59} = \frac{12}{59}$
 $\Rightarrow -40k = \ln\left(\frac{12}{59}\right)$ (1 mark) (4)

Given that 20 minutes after cooling began, the temperature of the water was 48°C

(b) evaluate the model.

(a) contd $\Rightarrow k = \frac{\ln\left(\frac{12}{59}\right)}{-40} = 0.03981\dots$
 $= 0.0398$ 3sf (1 mark) (2)

(c) Find, according to the model, the rate of the decrease in the temperature of the water 20 minutes after cooling began.

(Solutions relying entirely on calculator technology are not acceptable.)

(a) contd so $\theta = 21 + 59e^{-0.0398t}$ (1 mark) (3)

(b) After 20 minutes, model predicts

$$\theta = 21 + 59e^{-0.0398(20)} = 47.616\dots$$

$$= 47.6^\circ$$
 3sf (1 mark)

predicted actual
 $47.6 \approx 48$

so model seems appropriate (1 mark)

(c) $\frac{d\theta}{dt} = 59e^{-0.0398t} \times (-0.0398)$ by Chain Rule

$$= -2.3482e^{-0.0398t}$$
 (1 mark)

when $t=20$, $\frac{d\theta}{dt} = -2.3482e^{-0.0398(20)}$ (1 mark)

$$= -1.0593\dots$$

\Rightarrow decreasing at 1.06 3sf $^\circ\text{C}$ per minute (1 mark)