

10. A geometric series has

- first term $4 + k$
- second term $8 - 2k$

where k is a positive constant.

(a) Explain why the series does **not** converge when $k = 20$

(1)

Given that the series does converge,

(b) find the range of possible values of k .

(4)

(a) common ratio,

$$r = \frac{u_2}{u_1} = \frac{8-2k}{4+k}$$

$$\text{When } k=20, r = \frac{8-2(20)}{4+(20)} = \frac{-32}{24} = -1\frac{1}{3}$$

geometric series converge when $|r| < 1$, but $|-1\frac{1}{3}| = 1\frac{1}{3} > 1$
(1mark)

(b) Given convergence, $\left| \frac{8-2k}{4+k} \right| < 1$

$$\text{Given } k \text{ is positive, } |4+k| = 4+k \Rightarrow \frac{|8-2k|}{4+k} < 1$$

$$\text{Given } k \text{ is positive, } 4+k > 0 \Rightarrow |8-2k| < 4+k$$

(multiplying both sides by $4+k$
does not reverse inequality)

$$\text{Inequality is true when } 8-2k < 4+k \Rightarrow \frac{4}{3} < k \text{ (2 marks)}$$

$$\text{and when } -8+2k < 4+k \Rightarrow k < 12 \text{ (1mark)}$$

$$\text{so } \frac{4}{3} < k < 12 \text{ (1mark)}$$