

(2)

$$\int x^2 e^{x^3} dx$$

(b) Using the substitution  $u = x^3$  or otherwise, show by integration that

$$\int x^8 e^{x^3} dx = \frac{1}{3} e^{x^3} (x^6 + Ax^3 + B) + c$$

where  $A$  and  $B$  are constants to be found and  $c$  is an arbitrary constant.

(5)

(a) Differentiating an  $x^3$  term gives an  $x^2$  term by the Chain Rule, so try

$$y = e^{x^3} \Rightarrow \frac{dy}{dx} = e^{x^3} \times 3x^2 = 3x^2 e^{x^3}$$

this is 3 times too big, so  $\int x^2 e^{x^3} dx = \frac{1}{3} e^{x^3} + c$  (2 marks)

(b) Let  $u = x^3$

$$\frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$$

$$\int x^8 e^{x^3} dx = \int x^8 e^u \frac{du}{3x^2} = \frac{1}{3} \int x^6 e^u du$$

We want to get rid of  $x$  completely, and notice that  $x^6 = (x^3)^2 = u^2$ , so

$$\text{Integral} = \frac{1}{3} \int u^2 e^u du \quad (1 \text{ mark})$$

$$\begin{array}{ll} u = u^2 & v' = e^u \\ u' = 2u & v = e^u \end{array}$$

$$= \frac{1}{3} (u^2 e^u - \int 2u e^u) = \frac{1}{3} (u^2 e^u - 2 \int u e^u) \quad (2 \text{ marks})$$

$$\begin{array}{ll} u = u & v' = e^u \\ u' = 1 & v = e^u \end{array}$$

$$= \frac{1}{3} (u^2 e^u - 2(u e^u - \int 1 e^u)) \quad (1 \text{ mark})$$

$$= \frac{1}{3} (u^2 e^u - 2(u e^u - e^u)) + c = \frac{1}{3} u^2 e^u - \frac{2}{3} u e^u + \frac{2}{3} e^u + c$$

$$= \frac{1}{3} x^6 e^{x^3} - \frac{2}{3} x^3 e^{x^3} + \frac{2}{3} e^{x^3} = \frac{1}{3} e^{x^3} (x^6 - 2x^3 + 2) + c \quad (1 \text{ mark})$$

To handle the  $u^2$ , we now need to apply Integration by Parts twice.

$$\int u v' = uv - \int u'v$$