



14.

(a) $x = \frac{1}{2} \left(\frac{\pi}{3}\right) \sin\left(\frac{\pi}{3}\right)$
 $= \frac{1}{2} \left(\frac{\pi}{3}\right) \left(\frac{\sqrt{3}}{2}\right)$
 $= \frac{\pi\sqrt{3}}{12}$ (1 mark)

$y = \frac{1}{3} \sec\left(\frac{\pi}{3}\right)$
 $= \frac{1}{3} \left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right)$
 $= \frac{1}{3} \left(\frac{2}{1}\right) = \frac{2}{3}$ (1 mark)

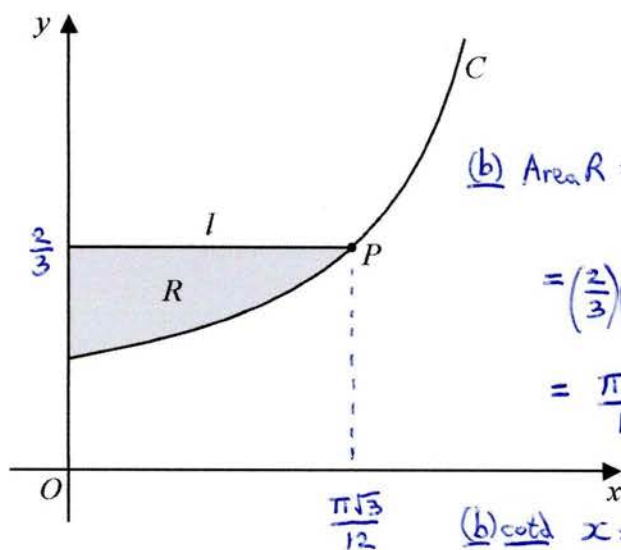


Figure 1

(b) Area R = Area under Line - Area under Curve
 $= \left(\frac{2}{3}\right) \left(\frac{\pi\sqrt{3}}{12}\right) - \int_0^{\frac{\pi\sqrt{3}}{12}} y \, dx$
 $= \frac{\pi\sqrt{3}}{18} - \int_0^{\frac{\pi\sqrt{3}}{12}} y \, dx$

(b) cotd $x = \frac{1}{2} \theta \sin \theta$
 $\frac{dx}{d\theta} = \frac{1}{2} \sin \theta + \frac{1}{2} \theta \cos \theta$ by Product Rule

Figure 1 shows a sketch of the curve C with parametric equations

$x = \frac{1}{2} \theta \sin \theta$ $y = \frac{1}{3} \sec \theta$ $0 \leq \theta \leq \frac{5\pi}{12}$

$\Rightarrow dx = \left(\frac{1}{2} \sin \theta + \frac{1}{2} \theta \cos \theta\right) d\theta$ (1 mark)

The point P lies on C where $\theta = \frac{\pi}{3}$

$\int_{x=0}^{\frac{\pi\sqrt{3}}{12}} y \, dx$
 $= \int_{\theta=0}^{\frac{\pi}{3}} \frac{1}{3} \sec \theta \left(\frac{1}{2} \sin \theta + \frac{1}{2} \theta \cos \theta\right) d\theta$ (2 marks)

For Limits

x	θ
$\frac{\pi\sqrt{3}}{12}$	$\frac{\pi}{3}$
0	0

(a) Find, in simplest form, the exact Cartesian coordinates of P

$= \int_0^{\frac{\pi}{3}} \frac{1}{3} \left(\frac{1}{\cos \theta}\right) \left(\frac{1}{2} \sin \theta + \frac{1}{2} \theta \cos \theta\right) d\theta$ (2)

The line l meets C at P and is parallel to the x-axis.

The region R shown shaded in Figure 1 is bounded by C, the y-axis and l.

(b) Show that the area of R is given by

$= \frac{1}{6} \int_0^{\frac{\pi}{3}} (\tan \theta + \theta) \, d\theta$

$A\pi\sqrt{3} - B \int_0^{\frac{\pi}{3}} (\tan \theta + \theta) \, d\theta$

so, $R = \frac{\pi\sqrt{3}}{18} - \frac{1}{6} \int_0^{\frac{\pi}{3}} (\tan \theta + \theta) \, d\theta$ (2 marks)

where A and B are rational numbers to be found.

(5)

(c) Hence, use algebraic integration to find the exact area of R

(3)

(c) $R = \frac{\pi\sqrt{3}}{18} - \frac{1}{6} \left[\ln |\sec \theta| + \frac{\theta^2}{2} \right]_0^{\frac{\pi}{3}}$ (1 mark)

$= \frac{\pi\sqrt{3}}{18} - \frac{1}{6} \left(\ln 2 + \frac{\left(\frac{\pi}{3}\right)^2}{2} - \ln 1 - 0 \right)$

$= \frac{\pi\sqrt{3}}{18} - \frac{1}{6} \left(\ln 2 + \frac{\pi^2}{18} - 0 - 0 \right)$ (1 mark)

$= \frac{\pi\sqrt{3}}{18} - \frac{1}{6} \ln 2 - \frac{\pi^2}{108}$ (1 mark)