14.

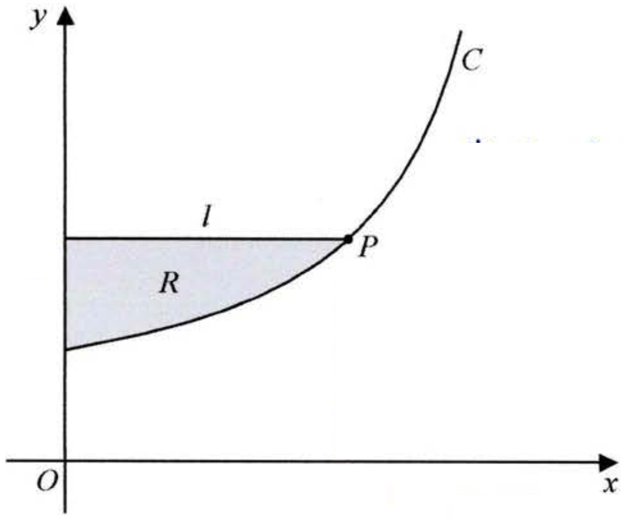

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \frac{1}{2}\theta\sin\theta \qquad y = \frac{1}{3}\sec\theta \qquad 0 \leqslant \theta \leqslant \frac{5\pi}{12}$$

The point P lies on C where $\theta = \frac{\pi}{3}$

(a) Find, in simplest form, the exact Cartesian coordinates of P

(2)

The line l meets C at P and is parallel to the x-axis.
The region R shown shaded in Figure 1 is bounded by C, the y-axis and l.

(b) Show that the area of R is given by

$$A\pi\sqrt{3} - B\int_0^{\frac{\pi}{3}} (\tan\theta + \theta)\,\mathrm{d}\theta$$

where A and B are rational numbers to be found.

(5)

(c) Hence, use algebraic integration to find the exact area of R

(3)