



Figure 2

Figure 2 shows a sketch of

- the curve  $C_1$  with equation  $y = \arccos \frac{1}{2}x$
- the curve  $C_2$  with equation  $y = \arcsin x$

The curves meet at the point  $P$  as shown in Figure 2.

(a) Show that at  $P$

$$\tan y = 2$$

(2)

(b) Hence, or otherwise, find the exact  $x$  coordinate of  $P$

(2)

$$\text{(a)} \quad y = \arccos\left(\frac{1}{2}x\right) \Rightarrow \frac{1}{2}x = \cos y$$

$$y = \arcsin x \Rightarrow x = \sin y \quad (1 \text{ mark})$$

$$\text{At intersection,} \quad \tan y = \frac{\sin y}{\cos y} = \frac{x}{\frac{1}{2}x} = 2 \quad (1 \text{ mark})$$

(b) Using identities

$$\sin^2 + \cos^2 = 1$$

$$\div \sin^2 \Rightarrow 1 + \cot^2 = \operatorname{cosec}^2$$

$$\div \cos^2 \Rightarrow \tan^2 + 1 = \sec^2$$

$$\tan^2 y = 4 \Rightarrow \sec^2 y - 1 = 4$$

$$\Rightarrow \sec^2 y = 5 \Rightarrow \sec y = \sqrt{5}$$

$$\Rightarrow \frac{1}{\cos y} = \sqrt{5} \Rightarrow \cos y = \frac{1}{\sqrt{5}}$$

$$\Rightarrow y = \arccos\left(\frac{1}{\sqrt{5}}\right) \Rightarrow \frac{1}{2}x = \frac{1}{\sqrt{5}}$$

$$\Rightarrow x = \frac{2}{\sqrt{5}} \quad (2 \text{ marks})$$