

| Question | Scheme | Marks | AOs |
|----------|--|-------|------|
| 2(a) | $(f(a) =) 4a^3 + 5a^2 - 10a + 4a = 0 \Rightarrow a(\dots) = 0$ | M1 | 3.1a |
| | $a(4a^2 + 5a - 6) = 0$ * | A1* | 1.1b |
| | | (2) | |
| (b)(i) | $a = \frac{3}{4}$ | B1 | 2.2a |
| (ii) | $4x^3 + 5x^2 - 10x + 4 \times \frac{3}{4} = 3 \Rightarrow 4x^3 + 5x^2 - 10x (= 0)$ | M1 | 1.1b |
| | $x = 0$ | B1 | 1.1b |
| | $x = \frac{-5 \pm \sqrt{185}}{8}$ | A1 | 1.1b |
| | | (4) | |

(6 marks)

Notes

(a)

M1: Attempts $f(a) = 0$ leading to an equation in a only and attempts to take a factor of a out. Condone slips. The $= 0$ may appear on a later line which is fine, but must be seen at some point in their solution in part (a).

A1*: Achieves the given answer with no errors including brackets.

Minimum acceptable is $4a^3 + 5a^2 - 10a + 4a = 0 \Rightarrow a(4a^2 + 5a - 6) = 0$

If the $= 0$ is absent at the start of their solution, it must appear before achieving the given answer.

Do not allow attempts to find the value of a and substitute that into $f(x)$

More difficult alternative methods may be seen:

Alt 1: You may see attempts via division / inspection

$$\begin{array}{r}
 4x^2 \quad + (5+4a)x \quad \quad \quad + (-10+5a+4a^2) \\
 x-a \overline{) 4x^3 \quad + 5x^2 \quad - 10x \quad + 4a} \\
 \underline{4x^3 \quad - 4ax^2} \\
 (5+4a)x^2 \quad - 10x \\
 \underline{(5+4a)x^2 \quad - a(5+4a)x} \\
 (-10+5a+4a^2)x \quad + 4a \\
 \underline{(-10+5a+4a^2)x - a(-10+5a+4a^2)} \\
 -6a+5a^2+4a^3
 \end{array}$$

Then sets remainder $-6a + 5a^2 + 4a^3 = 0$

M1: For dividing the cubic by $(x-a)$ leading to a quadratic quotient in x and a cubic remainder in a which is then set $= 0$ and attempts to take a factor of a out.

A1*: Completely correct with $a(4a^2 + 5a - 6) = 0$

Alt 2: You may also see a grid or an attempt at factorisation via inspection

| | | | |
|------|----------|--------------|-------------------|
| | $4x^2$ | $+(5+4a)x$ | $+(-10+5a+4a^2)$ |
| x | $4x^3$ | $+(5+4a)x^2$ | $(-10+5a+4a^2)x$ |
| $-a$ | $-4ax^2$ | $-a(5+4a)x$ | $-a(-10+5a+4a^2)$ |

OR $4x^3 + 5x^2 - 10x + 4a \equiv (x-a)(4x^2 + px - 4)$ which should be followed by equating the x terms and x^2 terms to form two equations which can be solved simultaneously.

$$-10 = -ap - 4 \quad \text{and} \quad 5 = -4a + p \Rightarrow p = 5 + 4a$$

$$\Rightarrow -10 = -a(5 + 4a) - 4 \Rightarrow 4a^2 + 5a - 6 = 0 \Rightarrow a(4a^2 + 5a - 6) = 0$$

The above are examples. There may be other correct attempts so look at what is done.

M1: For an attempt to set up two simultaneous equations by equating coefficients for x and equating coefficients for x^2 . Condone slips.

A1*: $4a^2 + 5a - 6 = 0 \Rightarrow a(4a^2 + 5a - 6) = 0$ Completely correct with no errors.

(b) Mark (i) and (ii) together

(i)

B1: Deduces that $a = \frac{3}{4}$ only. May be implied by their resultant cubic. If they do (b)(ii) multiple times using other roots for which $a \neq \frac{3}{4}$, then the solutions arising from using the other roots $a \neq \frac{3}{4}$ must be rejected

(ii)

M1: Attempts to substitute their $a = \frac{3}{4}$ (which must be positive) into $f(x)$, sets their $f(x) = 3$ and collects terms on one side ($= 0$ may be implied). Condone arithmetical and sign slips. Condone if they repeat this step using their other root(s).

B1: $(x =) 0$

A1: $(x =) \frac{-5 \pm \sqrt{185}}{8}$ (**and these values only**) or exact equivalent (ignore 0 for this mark). Withhold this mark if the fraction line was clearly not intended to be under both terms. This mark cannot be scored if they proceed directly to the roots from $4x^3 + 5x^2 - 10x$ without taking a factor of or dividing by x first to **see the quadratic factor**. Isw once the correct answers are seen if they proceed to provide rounded answers after.

e.g. $4x^3 + 5x^2 - 10x + 4 \times \frac{3}{4} = 3 \Rightarrow 0, \frac{-5 \pm \sqrt{185}}{8}$ is M0B1A0

e.g. $4x^3 + 5x^2 - 10x = 0 \Rightarrow 0, \frac{-5 \pm \sqrt{185}}{8}$ is M1B1A0

e.g. $4x^3 + 5x^2 - 10x = 0 \Rightarrow 4x^2 + 5x - 10 = 0 \Rightarrow \frac{-5 \pm \sqrt{185}}{8}$ is M1B0A1

e.g. $4x^3 + 5x^2 - 10x = 0 \Rightarrow x(4x^2 + 5x - 10) = 0 \Rightarrow 0, \frac{-5 \pm \sqrt{185}}{8}$ is M1B1A1