Question	Scheme	Marks	AOs
12	$\dfrac{\sin(x+h)-\sin x}{h}$	B1	2.1
	$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	M1 A1	1.1b 1.1b
	(As $h\to 0$), $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)\to 0\times\sin x+1\times\cos x$	dM1	2.1
	so $\dfrac{dy}{dx}=\cos x$ *	A1*	2.5
		(5 marks)	

Notes

Throughout the question allow the use of $h=\delta x$ if used consistently

There is no requirement to see "gradient of chord" written down.

B1: Gives the correct fraction such as $\dfrac{\sin(x+h)-\sin x}{x+h-x}$ or $\dfrac{\sin x-\sin(x+h)}{-h}$ or $\dfrac{\sin(x+h)-\sin(x-h)}{2h}$ or $\dfrac{\sin(x-h)-\sin x}{x-h-x}$. Condone invisible brackets. May be implied by $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

M1: Uses the compound angle formula for $\sin(x\pm h)$ to give $\sin x\cos h\pm\cos x\sin h$

A1: Achieves $\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$ or equivalent (may be implied by further work).
Allow invisible brackets to be recovered.

dM1: **It is dependent on both the B and the M marks being awarded.**
Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\dfrac{\cos h-1}{h}\right)$ and replace with 0 and isolate $\left(\dfrac{\sin h}{h}\right)$ and replace with 1.

e.g. $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\sin x\times 0+\cos x\times 1$

Accept as a minimum $\sin x\left(\dfrac{\cos h-1}{h}\right)+\cos x\left(\dfrac{\sin h}{h}\right)=\cos x$ (implying the application of the limits)

If they do not fully show $\left(\dfrac{\cos h-1}{h}\right)$ and $\left(\dfrac{\sin h}{h}\right)$ being isolated but proceed from

e.g. $\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$ to $0\times\sin x+\cos x$ (or e.g. $0+\cos x$) then this can be implied and score dM1

$\dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}=\cos x$ is dM0

Condone if limit notation remains within their expression after the limits have been applied.
e.g. $\lim\limits_{h\to 0}(\sin x\times 0+\cos x\times 1)$

Alternatively, condone use of the small angle approximations such that

$\dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}\to\dfrac{-\dfrac{h^2}{2}\sin x+h\cos x}{h}=-\dfrac{h}{2}\sin x+\cos x$ and replaces $\dfrac{h}{2}$ with 0

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x$ with no errors seen. (cso)

We need to see $h \to 0$ **at some point in their solution and linking** $\frac{\mathrm{d}y}{\mathrm{d}x}$ **with** $\cos x$ e.g.

- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \lim_{h\to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right) = \cos x$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \lim_{h\to 0}\left(-\frac{h}{2}\sin x + \cos x\right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)
- $\frac{\mathrm{d}y}{\mathrm{d}x} = ... = \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x$ as $h \to 0$

Condone $\mathrm{f}'(x)$ or y' in place of $\frac{\mathrm{d}y}{\mathrm{d}x}$

Give final A0 for no evidence of limiting arguments**:**

e.g. when $h = 0$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = ... = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \to 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x(\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.