Question	Scheme	Marks	AOs
14	When n is even: $$(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$$ $\Rightarrow$ which is odd **or** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	M1 A1	3.1a 2.2a
	When n is even: $(2k+1)^3-(2k)^3=8k^3+12k^2+6k+1-8k^3=6k(2k+1)+1$ $\Rightarrow$ which is odd **and** When n is odd: $(2k+2)^3-(2k+1)^3=8(k^3+3k^2+3k+1)-(8k^3+12k^2+6k+1)=6k(2k+3)+7$ $\Rightarrow$ which is odd	dM1	2.1
	Hence odd for all $n(\in \square)$ *	A1*	2.4
		(4 marks)	

Notes

General guidance

It is likely that you will see a mixture of methods and approaches within some solutions.

Mark the approach which scores the highest number of marks.

There should be no errors in the algebra but allow e.g. invisible brackets to be "recovered".

Withhold the final mark if n is used instead of k or reference to $n \in \square$ but $n \in \square^+$ is acceptable.

Main scheme algebraic method using e.g. $n=2k$ **and** $n=2k\pm1$

You will need to look at both cases and mark the one which is fully correct first.

Allow a different variable to k and may be different letters for odd and even

M1: For the key step attempting to find $(n+1)^3-n^3$ when $n=2k$ **or** $n=2k\pm1$ and attempting to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$ **or** $2n\pm5$)
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1: Complete argument for $n=2k$ **or** $n=2k+1$ (or e.g. $n=2k-1$) showing the result is odd.
Requires:

- Correct simplified quadratic expression e.g. $12k^2+6k+1$ (when $n=2k$), $12k^2+18k+7$ (when $n=2k+1$), $12k^2-6k+1$ (when $n=2k-1$) (may be factorised)
- A reason why the expression is odd e.g. $2k(6k+3)+1$ or may use a divisibility argument e.g.
$$\frac{12k^2+6k+1}{2}=6k^2+3k+\frac{1}{2}$$
- Concludes "odd" o.e. (may be within their final conclusion)
There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered"
Condone the use of e.g. $n=2n$ and $n=2n\pm1$

dM1: Attempts to find $(n+1)^3-n^3$ when $n=2k$ **and** $n=2k\pm1$ and attempts to multiply out and simplify to achieve a three term quadratic (allow equivalent representation of odd or even e.g. $n=2k+2$, $2n\pm5$
Condone arithmetical slips and condone the use of e.g. $n=2n$ and $n=2n\pm1$

A1*: Complete argument for **both** $n=2k$ **and** $n=2k+1$ (or e.g. $n=2k-1$) showing the result is odd for all $n(\in\mathbb{Z})$

Requires for both cases:

- Correct simplified expressions for both odd and even (which may be factorised)
- A reason why both of the expressions are odd
- Minimal conclusion (may be within their final conclusion)

An overall conclusion is also required. "Hence odd for all $n(\in\mathbb{Z})$" Accept "hence proven", "statement proved", "QED"

The conclusion for when $n=2k$ and $n=2k+1$ may be within the final conclusion rather than separate which is acceptable e.g. "when $n=2k$ and when $n=2k+1$ the expression is odd, hence proven" (following correct simplified expressions and reasons)

	$(n+1)^3$	n^3	$(n+1)^3-n^3$
$n=2k-1$	$8k^3$	$8k^3-12k^2+6k-1$	$12k^2-6k+1$
$n=2k$	$8k^3+12k^2+6k+1$	$8k^3$	$12k^2+6k+1$
$n=2k+1$	$8k^3+24k^2+24k+8$	$8k^3+12k^2+6k+1$	$12k^2+18k+7$

Alternative methods:

Algebraic with logic example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic. Condone arithmetical slips.

A1: Correct quadratic expression $3n^2+3n+1$

dM1: Attempts to factorise their quadratic such that $n^2+n\to n(n+1)$ within their expression e.g. $3n(n+1)+1$

A1*: Explains that e.g. $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even so $3n(n+1)+1$ is odd hence odd for all $n(\in\mathbb{Z})$

Proof by contradiction example

M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic.

A1: Correct quadratic expression $3n^2+3n+1$

dM1: Sets $3n^2+3n+1=2k$ (for some integer k) $\Rightarrow 3n(n+1)=2k-1$

A1*: Explains that $n(n+1)$ is always even as it is the product of two consecutive numbers so $3n(n+1)$ is odd $\times$ even $=$ even but $2k-1$ is odd hence we have a contradiction so $(n+1)^3-n^3$ is odd (for all $n(\in\mathbb{Z})$. There must have been a correct opening statement setting up the contradiction e.g. "assume that there exists a value for n for which $(n+1)^3-n^3$ is even"

Solutions via just logic (no algebraic manipulation)

e.g.

If n is odd, then $(n+1)^3 - n^3$ is $\text{even}^3 - \text{odd}^3 = \text{even} - \text{odd} = \text{odd}$

If n is even, then $(n+1)^3 - n^3$ is $\text{odd}^3 - \text{even}^3 = \text{odd} - \text{even} = \text{odd}$

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for $n=k$, substitutes $n=k+1$ into $(n+1)^3 - n^3$, multiplies out the brackets and attempts to simplify to a three term quadratic e.g. $3k^2+9k+7$ Condone arithmetical slips

A1: $\left(\mathrm{f}(k+1) = 3k^2+3k+1+6(k+1) =\right)$ $(k+1)^3 - k^3 + 6(k+1) = \mathrm{f}(k) + 6(k+1)$ which is odd + even = odd

dM1: Attempts to substitute $n=1 \Rightarrow (1+1)^3 - 1^3 = 7$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n=1$
- if it is true for $n=k$ then it is true for $n=k+1$
- therefore it is true for all $n\left(\in \mathbb{N}\right)$