

3. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\vec{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of a for which

$$|\vec{OB}| > |\vec{OA}|$$

(2)

(a) $\vec{OA} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$ $|\vec{OA}| = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38}$

(b) $\vec{OB} = \begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$ $|\vec{OB}| = \sqrt{2^2 + 4^2 + a^2} = \sqrt{20 + a^2}$

$$|\vec{OB}| > |\vec{OA}| \Rightarrow \sqrt{20 + a^2} > \sqrt{38}$$

$$20 + a^2 > 38 \quad \left(\begin{array}{l} \text{both sides are magnitudes} \\ \therefore > 0 \end{array} \right)$$

$$a^2 > 18$$

Given a is positive integer,

$$a^2 = 25 \quad \& \quad a = 5$$