

Solutions relying entirely on calculator technology are not acceptable.

The curve  $C$  has equation  $y = f(x)$  where  $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \frac{1}{2} \cos x$
- the curve has a stationary point with  $x$  coordinate  $\alpha$
- $\alpha$  is small

(a) use the small angle approximation for  $\cos x$  to estimate the value of  $\alpha$  to 3 decimal places.

(3)

The point  $P(0, 3)$  lies on  $C$

(b) Find the equation of the tangent to the curve at  $P$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are constants to be found.

(2)

$$(a) \quad f'(\alpha) = 0$$

$$2\alpha + \frac{1}{2} \cos \alpha = 0$$

using small angle approx,

$$2\alpha + \frac{1}{2} \left(1 - \frac{\alpha^2}{2}\right) \approx 0$$

$$2\alpha + \frac{1}{2} - \frac{\alpha^2}{4} \approx 0$$

$$8\alpha + 2 - \alpha^2 \approx 0$$

$$\alpha^2 - 8\alpha - 2 \approx 0$$

$$\alpha \approx 4 \pm 3\sqrt{2} \approx 8.2426\dots, -0.2426\dots$$

Given  $\alpha$  is small,  $\alpha = -0.243$  (3dp)

(b) At  $P(0, 3)$ , gradient  $m = 2(0) + \frac{1}{2} \cos(0) = \frac{1}{2}$

Tangent is  $(y-3) = \frac{1}{2}(x-0)$

$$\Rightarrow y = \frac{1}{2}x + 3$$