

5. A continuous curve has equation $y = f(x)$.

The table shows corresponding values of x and y for this curve, where a and b are constants.

x	3	3.2	3.4	3.6	3.8	4
y	a	16.8	b	20.2	18.7	13.5

The trapezium rule is used, with all the y values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(a) show that $a + 2b = 51$

(3)

Given also that the sum of all the y values in the table is 97.2

(b) find the value of a and the value of b

(3)

$$\begin{aligned} \underline{\text{(a)}} \quad \int &\approx \frac{1}{2} \left(\frac{4-3}{5} \right) (a + 2(16.8 + b + 20.2 + 18.7) + 13.5) \\ &\approx \frac{1}{10} (a + 2b + 124.9) = 17.59 \quad \leftarrow \text{Given} \end{aligned}$$

$$a + 2b + 124.9 = 175.9$$

$$a + 2b = 175.9 - 124.9 = 51$$

$$\underline{\text{(b)}} \quad a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2$$

$$a + b = 97.2 - 16.8 - 20.2 - 18.7 - 13.5 = 28$$

$$\begin{array}{r} a + 2b = 51 \\ -(a + b = 28) \\ \hline \end{array}$$

$$b = 23$$

$$a = 28 - 23 = 5$$