

7. The function f is defined by

$$f(x) = 3 + \sqrt{x-2} \quad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x-3} \quad x \in \mathbb{R} \quad x \neq 3$$

(c) Find $gf(6)$

(2)

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

(a) $x > 2 \Rightarrow \sqrt{x-2} > 0 \Rightarrow 3 + \sqrt{x-2} > 3 \Rightarrow f(x) > 3$

(b) let $y = 3 + \sqrt{x-2}$ then $y - 3 = \sqrt{x-2}$

$$y^2 - 6y + 9 = x - 2 \Rightarrow x = y^2 - 6y + 11$$

so $f^{-1}(x) = x^2 - 6x + 11$

(c) $f(6) = 3 + \sqrt{6-2} = 5$

$$gf(6) = g(5) = \frac{15}{5-3} = 7\frac{1}{2}$$

(d) $f(a^2+2) = 3 + \sqrt{a^2+2-2} = 3+a$

$$g(a) = \frac{15}{a-3}$$

$$f(a^2+2) = g(a) \Rightarrow 3+a = \frac{15}{a-3} \Rightarrow (3+a)(a-3) = 15$$

$$\Rightarrow a^2 - 9 = 15 \Rightarrow a^2 = 24 \Rightarrow a = \sqrt{24} = 2\sqrt{6}$$