

9. The first three terms of a geometric sequence are

$$3k+4 \quad 12-3k \quad k+16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0 \quad (2)$$

Given that the sequence converges,

(b) (i) find the value of k , giving a reason for your answer,

(ii) find the value of S_{∞} .

(5)

$$(a) \quad \frac{12-3k}{3k+4} = \frac{k+16}{12-3k} \Rightarrow (12-3k)(12-3k) = (k+16)(3k+4)$$

$$\Rightarrow 144 - 72k + 9k^2 = 3k^2 + 52k + 64$$

$$\Rightarrow 6k^2 - 124k + 80 = 0$$

$$\Rightarrow 3k^2 - 62k + 40 = 0$$

$$(b) (i) \quad 3k^2 - 60k - 2k + 40 = 0$$

$$(3k - 2)(k - 20) = 0$$

$$k = \frac{2}{3}, 20$$

$$r = \frac{12 - 3(\frac{2}{3})}{3(\frac{2}{3}) + 4} \quad \text{or} \quad \frac{12 - 3(20)}{3(20) + 4}$$

$$= \frac{10}{6} \quad \text{or} \quad \frac{-48}{64} = 1\frac{2}{3} \quad \text{or} \quad -\frac{3}{4}$$

for convergence, $|r| < 1$, so $r = -\frac{3}{4}$ with $k = 20$

$$(b) (ii) \quad S_{\infty} = \frac{a}{1-r} \quad a = 3(20) + 4 = 64$$

$$S_{\infty} = \frac{64}{1 - (-\frac{3}{4})} = 36\frac{4}{7}$$