

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k ,

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k .

(6)

$$\begin{aligned} \text{(a)} \quad x^2 + 6kx + \left(\frac{6k}{2}\right)^2 - \left(\frac{6k}{2}\right)^2 + y^2 - 2ky + \left(\frac{2k}{2}\right)^2 - \left(\frac{2k}{2}\right)^2 + 7 = 0 \\ \underbrace{(x+3k)^2 - 9k^2}_{(x+3k)^2 - 9k^2} + \underbrace{y^2 - 2ky + k^2 - k^2}_{(y-k)^2 - k^2} + 7 = 0 \end{aligned}$$

$$(x+3k)^2 + (y-k)^2 = 10k^2 - 7$$

$$\text{(a)(i) Centre: } (-3k, k) \quad \text{(a)(ii) Radius: } \sqrt{10k^2 - 7}$$

(b) At intersection,

$$(x+3k)^2 + (2x-1-k)^2 = 10k^2 - 7$$

$$x^2 + 6kx + 9k^2 + 4x^2 - 2x - 2kx - 2x + 1 + k - 2kx + k + k^2 = 10k^2 - 7$$

$$5x^2 + (2k-4)x + (2k+8) = 0$$

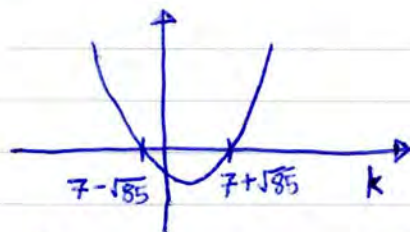
For 2 distinct solutions, discriminant, $b^2 - 4ac > 0$

$$(2k-4)^2 - 4(5)(2k+8) > 0$$

$$4k^2 - 16k + 16 - 40k - 160 > 0$$

$$4k^2 - 56k - 144 > 0$$

$$k^2 - 14k - 36 > 0$$



From sketch, $\{k < 7 - \sqrt{85}\} \cup \{k > 7 + \sqrt{85}\}$