

12.

$$y = \sin x$$

where  $x$  is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for  $\sin(A \pm B)$
- assume that as  $h \rightarrow 0$ ,  $\frac{\sin h}{h} \rightarrow 1$  and  $\frac{\cos h - 1}{h} \rightarrow 0$

(5)

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \quad (\sin(A+B) = \sin A \cos B + \sin B \cos A)$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x) \cos(h) + \sin(h) \cos(x) - \sin(x)}{h}$$

$$= \lim_{h \rightarrow 0} \left( \frac{\sin(x) (\cos(h) - 1)}{h} + \frac{\cos(x) \sin(h)}{h} \right)$$

$$= \sin(x) (0) + \cos(x) (1)$$

$$= \cos(x)$$