

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height,  $H$  m, of a carriage above the ground,  $t$  seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

(d) First model would give negative ever-decreasing  $H$  as  $t \rightarrow \infty$   
Second model would cycle.

where  $a$  and  $b$  are positive constants.

(e) Max  $H = 60$   
when  $\cos(9t + \alpha) = 1$   
 $\Rightarrow 60 = 29(1) + \beta$   
 $\Rightarrow \beta = 31$

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when  $t = 40$

(1)

In an alternative model, the vertical height,  $H$  m, of a carriage above the ground,  $t$  seconds after the carriage starts the first circuit, is given by

$$H = 29 \cos(9t + \alpha)^\circ + \beta \quad 0 \leq \alpha < 360^\circ$$

where  $\alpha$  and  $\beta$  are constants.

(c) Find a complete equation for the alternative model.

(e) contd When  $t=0, H=2$ , so  
 $2 = 29 \cos(\alpha) + 31$   
 $\cos(\alpha) = -1$   
 $\Rightarrow \alpha = 180^\circ$

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

(a) When  $t=0, H=2$ , so  $2 = a - b(-20)^2$   
 $2 = a - 400b$

Max  $H$  is when  $(t-20)^2 = 0 \Rightarrow t = 20$   
so,  $60 = a - b(0)^2 = a$

$$2 = 60 - 400b \Rightarrow b = \frac{-58}{-400} = 0.145$$

$$H = 60 - 0.145(t - 20)^2$$

(b) When  $t = 40, H = 60 - 0.145(40 - 20)^2 = 2$  m.