

14. Prove, using algebra, that

$$(n+1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

$$(n+1)^3 - n^3 = n^3 + 3n^2 + 3n + 1 - n^3$$

$$= 3n^2 + 3n + 1$$

$$= 3n(n+1) + 1$$

either n is even or $(n+1)$ is even,
so $3n(n+1)$ is even

$$3n(n+1) + 1 = \text{even} + 1 = \text{odd}$$