Question	Scheme	Marks	AOs
14(a)	$\frac{\mathrm{d}u}{\mathrm{d}x} = 3^x \ln 3$	B1	1.1b
	$\int_{0}^{2} \frac{3^{x}}{\left(1+3^{x}\right)^{3}} dx = \int \frac{u}{\left(1+u\right)^{3}} \frac{1}{u \ln 3} du$	M1	1.1b
	$\int_{0}^{2} \frac{3^{x}}{\left(1+3^{x}\right)^{3}} dx = \frac{1}{\ln 3} \int_{1}^{9} \frac{1}{\left(1+u\right)^{3}} du$	A1	2.1
		(3)	
(b)	$\int_{0}^{2} \frac{3^{x+2}}{\left(1+3^{x}\right)^{3}} dx = 9 \int_{0}^{2} \frac{3^{x}}{\left(1+3^{x}\right)^{3}} dx \text{ or } \frac{9}{\ln 3} \int_{1}^{9} \frac{1}{\left(1+u\right)^{3}} du$	B1	3.1a
	$\int (1+u)^{-3} du \to(1+u)^{-2}$	M1	1.1b
	$-\frac{k}{2}(1+u)^{-2}$	A1	1.1b
	$\left(-\frac{9}{2\ln 3}(1+9)^{-2}\right) - \left(-\frac{9}{2\ln 3}(1+1)^{-2}\right)$	dM1	1.1b
	$\frac{27}{25\ln 3}$	A1	2.1
		(5)	
(8 marks)			

Notes:

(a)

B1: Uses the chain rule correctly to find $\frac{du}{dx} = 3^x \ln 3$

M1: Attempts a complete substitution from x to u ignoring limits. dx must be replaced with du and

 3^x must be replaced with *u*. Condone slips in finding and rearranging $\frac{du}{dx}$ but the '*u*'s must be able to cancel to achieve the given form.

A1: cso
$$\int_{0}^{2} \frac{3^{x}}{(1+3^{x})^{3}} dx = \frac{1}{\ln 3} \int_{1}^{9} \frac{1}{(1+u)^{3}} du$$
 with no errors seen and including limits.

(b)

B1: Attempts to factor out 3^2 or 9 from the integral which may be implied e.g. by

 $\frac{9}{\ln 3} \int_{1}^{9} \frac{1}{\left(1+u\right)^3} \mathrm{d}u$

M1: Writes ... $\int \frac{1}{(1+u)^3} du$ as ... $\int (1+u)^{-3} du$ and integrates, increasing the power by 1 to $...(1+u)^{-2}$ A1: Correct integration of $k \int \frac{1}{(1+u)^3} du$ to $-\frac{k}{2}(1+u)^{-2}$ Note that the correct expression is $-\frac{9}{2 \ln 3} (1+u)^{-2}$ **dM1:** Substitutes in their limits from (a) into an expression of the correct form $...(1+u)^{-2}$ and

subtracts either way round.

A1: cso A rigorous mathematical argument leading to $\frac{27}{25 \ln 3}$