

Question	Scheme	Marks	AOs
7(a)	$\{H =\} 0.6e^{-0.2t} \{+c\}$	M1	1.1b
	$t = 0, H = 1.5 \Rightarrow 1.5 = "0.6" + c$ $\Rightarrow c = 0.9$	dM1	3.4
	$\Rightarrow H = 0.6e^{-0.2t} + 0.9$	A1	2.1
	(3)		
(b)	$1.2 = 0.6e^{-0.2t} + 0.9 \Rightarrow 0.6e^{-0.2t} = 0.3$	M1	3.4
	$e^{-0.2t} = \frac{1}{2}$ $\Rightarrow t = -5 \ln\left(\frac{1}{2}\right)$	dM1	1.1b
	$\{t =\} 3 \text{ hours } 28 \text{ minutes}$	A1	3.2a
	(3)		
(c)	$\{\text{As } t \text{ gets large } H \rightarrow\} 0.9$	M1	3.1b
	0.9 m or 90 cm	A1ft	2.2b
	(2)		

(8 marks)

Notes

(a)

M1: Attempts integration to achieve $\{H =\} k e^{-0.2t} \{+c\}$ with k a numerical constant $\neq -0.12$
Note that we will condone $k = (-0.2)(-0.12) \{= 0.024\}$

If they divide by -0.12 first before integrating they need $aH = be^{-0.2t} \{+c\}$ with a and b numerical and $b \neq 1$. Condone a spurious integral symbol remaining after integration.

dM1: Uses $t = 0, H = 1.5$ and a model of the form $H = k e^{-0.2t} + c$ (or $aH = be^{-0.2t} + c$) to find the value of the constant c . They cannot just “make up” a value for k .

Do not be concerned with their processing to find c but they cannot just state B (or c) is 0.

For reference if they divide by -0.12 first they should reach $-\frac{25}{3} H = -5e^{-0.2t} - 7.5$ o.e.

A1: Correct complete equation in the required form: $H = 0.6e^{-0.2t} + 0.9$ with the $H =$ present.

May be awarded if seen at the start of (b) but not in (c). Condone $-\frac{1}{5}$ in place of -0.2

Finding correct values for A and B is insufficient for this mark.

Allow exact equivalents but they must be in the required form, e.g. $H = \frac{6}{10} e^{-0.2t} + \frac{9}{10}$

A minimally acceptable answer is $\{H =\} 0.6e^{-0.2t} + c \rightarrow H = 0.6e^{-0.2t} + 0.9$ score M1dM1A1.

Note: sight of differentiating the given form to e.g. $\frac{dH}{dt} = -0.2Ae^{-0.2t}$ in their working

without clear evidence of integration of the original differential equation should be marked using the special case below.

SC: For candidates starting with the given answer $H = Ae^{-0.2t} + B$ it is possible to use

$\frac{dH}{dt} = -0.2Ae^{-0.2t} = -0.12e^{-0.2t}$ to deduce that $A = 0.6$. This can be awarded SC M1dM0A0

If they go on to find B as in the main scheme then this can be awarded SC M1dM1A0

Answer with no working scores 110.

Note: If the special case is applied they may go on to achieve the rest of the marks in (b) and (c).

(b) Note: A and B must be numbers but may be “made up” if they did not have an answer to (a).

M1: Uses $H = 1.2$ in a model of the form $H = Ae^{-0.2t} + B$, $B \neq 0$ and rearranges to make $Ae^{\pm 0.2t}$ or $e^{\pm 0.2t}$ the subject. Condone slips in rearranging, e.g. dividing the LHS by 0.9 instead of subtracting 0.9. Rearranging first before substituting is acceptable but they must get to $Ae^{\pm 0.2t}$ or $e^{\pm 0.2t}$ as the subject.

dM1: Correct use of \ln to make t the subject. Requires $A > 0$, $0 < B < 1.2$ **and** $e^{\pm 0.2t} = \lambda > 0$. If they had a negative value for A in part (a) they cannot just make it positive at this stage.

Any of $5 \ln 2$ or $-5 \ln \frac{1}{2}$ or awrt 3.46 or awrt 3.47 following a correct equation will imply

M1dM1.

If they do not show their method for an incorrect $H = Ae^{-0.2t} + B$ with $A > 0$, $0 < B < 1.2$ you may need to check their value for $t > 0$ as it may imply M1dM1.

A1: Correct time in hours and minutes $\{t = \}$ 3 hours 28 minutes, but condone e.g. 3h 28m

Must come from correct values of A and B in (a).

Note: If their $B = 0$ then they should end up with $t = -3.46\dots$ however, they did not score the first M1. They cannot “recover” this by making it positive and finding $t = 3$ hours 28 minutes.

(c) Note that 0.9 or 0.9m must come from a correct value of B in (a) to score any marks.

M1: Identifies the requirement to establish the limit as t tends to infinity.

It can be implied by stating that $H = Ae^{-0.2t} + B \rightarrow B$ or $\left(\lim_{t \rightarrow \infty} [0.6e^{-0.2t} + 0.9]\right) = 0.9$

Stating “ B ” on its own will score this mark.

Substituting a large value is M0 unless it leads to their value for B at which point the A1 is available as well.

A1ft: Correct height including units. Follow through on their value of B where $0 < B < 1.2$

Correct ft height including units implies M1A1, while e.g. 0.9 (no units) would imply M1A0.

Evidence of an incorrect method such as $1.5 - 0.6e^{-0.2(0)} = 0.9$ m scores M0A0.

Misreading as $\frac{dH}{dt} = -0.12e^{0.2t}$ can score a maximum (a) 110 (b) 100 (c) 10.