Question	Scheme		Marks	AOs
9(a)	$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}\Rightarrow\frac{3^{2(7-2k)}}{3^{4k-5}}=\frac{3^{2(k-1)}}{3^{2(7-2k)}}$		M1	3.1a
	$\left(3^{2(7-2k)}\right)^2=3^{4k-5}\times3^{2(k-1)}$ $\Rightarrow 28-8k=6k-7\Rightarrow k=...$	$3^{2(7-2k)-(4k-5)}=3^{2(k-1)-2(7-2k)}$ $\Rightarrow 19-8k=6k-16\Rightarrow k=...$	dM1	1.1b
	$k=\frac{5}{2}$ *		A1*	2.1
			(3)	
(b)	$a=3^{4(2.5)-5}$ and $r=\frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}\Rightarrow$ one of $a=243$ or $r=\frac{1}{3}$		M1	2.2a
	$S_\infty=\frac{a}{1-r}=\frac{"243"}{1-"\frac{1}{3}"}$		M1	1.1b
	$S_\infty=\frac{729}{2}\,(364.5)$ cao		A1	1.1b
			(3)	
				(6 marks)

Notes

(a) **Special cases:**

SC 1: For those that verify rather than prove a SC 100 is awarded for substituting $k=\frac{5}{2}$ into all three terms to correctly obtain 243, 81 and 27 **with** a statement that this is **geometric** with $r=\frac{1}{3}$ (or equivalent reason). All statements must be correct.

SC 2: Be aware that e.g. $\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}\Rightarrow 81^{2(7-2k)}=9^{6k-7}$ is an incorrect process (without some indication that they have intentionally squared both sides) that fortuitously leads to the correct answer and may score maximum SC 010.

M1: Uses the 3 terms to set up an equation in k **and**

- **either** reaches a common base by replacing 9 with 3^2 or by replacing 3 with $9^{0.5}$ and uses the power law of indices correctly
- **or** uses the laws of indices correctly to reach $9^{14-4k}=3^{6k-7}$ condoning slips in e.g. expanding brackets.

Writing down e.g. $2(7-2k)-(4k-5)=2(k-1)-2(7-2k)$ is sufficient to imply the M1.

dM1: Correct processing leading to a value for k.

A1*: Correct value following correct working. Allow $k=2.5$ in place of $k=\frac{5}{2}$

Condone missing/invisible brackets provided they are recovered correctly.

Alt 1 Using Base 9:

$$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}\Rightarrow\frac{9^{7-2k}}{9^{2k-2.5}}=\frac{9^{k-1}}{9^{7-2k}}\text{ o.e. scores M1}$$

$$\Rightarrow 9^{9.5-4k}=9^{3k-8}\Rightarrow 9.5-4k=3k-8\Rightarrow 7k=17.5\Rightarrow k=2.5$$

Score dM1 when a value of k is achieved using a correct process and A1* if fully correct.

Alt 2 Finding *r* in terms of *k* and using e.g. $u_3 = ar^2$:

$$r = \frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(7-2k)}}{3^{4k-5}}\left\{= 3^{19-8k}\right\} \text{ or } r = \frac{3^{2(k-1)}}{9^{7-2k}} = \frac{3^{2k-2}}{3^{2(7-2k)}}\left\{= 3^{6k-16}\right\}$$

$$\Rightarrow 3^{4k-5}\times\left(3^{19-8k}\right)^2 = 3^{2k-2} \text{ or } \Rightarrow 3^{4k-5}\times\left(3^{6k-16}\right)^2 = 3^{2k-2} \text{ scores M1}$$

$$\Rightarrow 3^{4k-5}\times 3^{2(19-8k)} = 3^{2k-2} \Rightarrow 33-12k = 2k-2 \Rightarrow 14k = 35 \Rightarrow k = 2.5$$

Score dM1 when a value of *k* is achieved using a correct process and A1* if fully correct.

Alt 3 Using Logs Way 1:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \left(9^{7-2k}\right)^2 = 3^{6k-7} \Rightarrow 9^{14-4k} = 3^{6k-7} \text{ scores M1}$$

$$\Rightarrow (14-4k)\log_3 9 = 6k-7$$

$$\Rightarrow 2(14-4k) = 6k-7$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of *k* is achieved using a correct process and A1* if fully correct.

Alt 4 Using Logs Way 2:

$$\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}}$$

$$\Rightarrow (7-2k)\log_3 9-(4k-5)\log_3 3 = (2k-2)\log_3 3-(7-2k)\log_3 9 \quad \text{scores M1}$$

$$\Rightarrow 2(7-2k)-(4k-5) = 2k-2-2(7-2k)$$

$$\Rightarrow 19-8k = 6k-16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of *k* is achieved using a correct process and A1* if fully correct.

Alt 5 Recognising that taking $\log_3$ forms an Arithmetic Sequence:

$$\{\log_3\}u_1 = 4k-5,\ \{\log_3\}u_2 = 2(7-2k),\ \{\log_3\}u_3 = 2(k-1)$$

$$\Rightarrow 2(7-2k)-(4k-5) = 2(k-1)-2(7-2k) \quad \text{scores M1}$$

$$\Rightarrow 19-8k = 6k-16$$

$$\Rightarrow k = 2.5$$

Score dM1 when a value of *k* is achieved using a correct process and A1* if fully correct.
There is no need to see any mention of log in this approach.

(b)

M1: Deduces expressions for the first term **and** the common ratio using $k = \frac{5}{2}$ in the <u>correct formulae</u> **and** finds at least one of *a* = 243 or $r = \frac{1}{3}$. Allow if seen in (a). May be implied by correct values for *a* and *r*. For reference, $a = 3^{4(2.5)-5}$ and $r = \frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}$ $\left\{\text{or } r = \frac{3^{2(2.5-1)}}{9^{7-2(2.5)}}\right\}$

M1: Recalls the sum to infinity formula and substitutes their values for *a* and *r* provided $|r| < 1$
Dependent on a correct attempt to find both *a* and *r* using *k* = 2.5 but allow if neither value is correct or if they are unprocessed e.g. $\dfrac{3^{4(2.5)-5}}{1-\dfrac{9^{7-2(2.5)}}{3^{4(2.5)-5}}}$ scores this mark.

A1: cao. Correct sum to infinity. Answer only (with no working) scores full marks. Apply isw.