Question	Scheme	Marks	AOs
14(a)	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{\sqrt{r}}$	B1	3.3
		(1)	
(b)	$0.9 = \frac{k}{\sqrt{16}} \Rightarrow k = 3.6$	B1	3.4
	$\int \sqrt{r}\ \mathrm{d}r = \int \text{"3.6"} \mathrm{d}t \Rightarrow ...$	M1	2.1
	$\frac{2}{3} r^{\frac{3}{2}} = \text{"3.6"}t \quad \{+c\}$	A1	1.1b
	$t = 10, r = 16 \Rightarrow \frac{2}{3} \times 16^{\frac{3}{2}} = 3.6 \times 10 + c \Rightarrow c = ...$	dM1	3.4
	$r^{\frac{3}{2}} = 5.4t + 10$ *	A1*	1.1b
		(5)	
(c)	$t = 20 \Rightarrow r = \left(5.4(20) + 10\right)^{\frac{2}{3}} = ...$	M1	3.4
	$r = 24.1$ cm	A1	1.1b
		(2)	
(d)	(The model will not hold indefinitely as) the balloon may burst	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

B1: Correctly sets up the model. $\frac{\mathrm{d}r}{\mathrm{d}t} \propto \frac{1}{\sqrt{r}}$ scores B0 unless e.g. $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{\sqrt{r}}$ is seen but condone $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{\sqrt{r}}$ being seen at the start of (b). They may use any letter except *t* or *r* in place of *k*.

You may see $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{1}{k\sqrt{r}}$ which is acceptable provided it is clear that it is not the k^{th} root.

(b) **Note:** candidates using $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{1}{\sqrt{r}}$ in (b) can score maximum B0M1A1dM0A0

B1: $k = 3.6$ coming from $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{\sqrt{r}}$ (or $k = \frac{5}{18}$ coming from $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{k\sqrt{r}}$) and from use of $r = 16$ and $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.9$ but note that this may occur later in their working, which is perfectly fine provided it is from acceptable work. Note e.g. $k = -3.6$ coming from $\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{\sqrt{r}}$ is correct.

Note, however, that an attempt to find *k* from comparing coefficients between $r^{\frac{3}{2}} = 1.5kt + c$ and $r^{\frac{3}{2}} = 5.4t + 10$ is not acceptable (see special case).

They can also find *k* by differentiating **their** $r = \mathrm{f}(t)$ and substituting $t = 10$, $r = 16$ and $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.9$ **if** they have also used $t = 10$, $r = 16$ in **their** $r = \mathrm{f}(t)$. This sets up simultaneous equations where *c* can be eliminated. Use Review if you are unsure if their approach is acceptable.

M1: Separates the variables for their differential equation **correctly** and attempts to integrate both sides.

Must be a differential equation of the form $\frac{\mathrm{d}r}{\mathrm{d}t} = \mathrm{f}(r)$ for some function f(r) independent of t.

Evidence of $r^n \to r^{n+1}$, or e.g. $\frac{1}{r} \to \ln r$ is sufficient for their attempt to integrate in r, but k must be integrated to kt o.e. e.g. $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^2} \Rightarrow r^2 \frac{\mathrm{d}r}{\mathrm{d}t} = k \Rightarrow \lambda r^3 = kt\{+c\}$ would score this mark.

Note that they may divide by k (or 3.6) prior to integrating. Here, 1 must be integrated to t.

A1: Correct integration for their k. Allow this mark if they have not found k, so allow e.g. $\frac{2}{3}r^{\frac{3}{2}} = kt\{+c\}$ with/without the constant of integration but the $\frac{2}{3}$ must be evident in some way.
Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.

dM1: Uses $t = 10$, $r = 16$ in their equation to find the constant of integration.
This mark is dependent on the first method mark.
Must have already found a value for k using a valid strategy and the constant of integration must be present.

Those that found k from comparing coefficients between $r^{\frac{3}{2}} = \frac{3}{2}kt + c$ and $r^{\frac{3}{2}} = 5.4t + 10$ may not score this mark.

A1*: Correct equation from correct working. May be seen at the start of (c).
Must follow A1 earlier so do check if this has been obtained fortuitously.

SC: It is possible to compare coefficients (following integration) between $r^{\frac{3}{2}} = \frac{3}{2}kt + c$ and $r^{\frac{3}{2}} = 5.4t + 10$ to deduce the value of k as 3.6 (or just write their coefficient as 5.4).
The maximum that can be scored this way or by a similar invalid approach is B0M1A1dM0A0

(c)

M1: Substitutes $t = 20$ into the given equation and uses correct processing to find the value of r

e.g. substitutes $t = 20$ into $\frac{2}{3}r^{\frac{3}{2}} = 72 + \frac{20}{3} \Rightarrow r = \left(\frac{3}{2}\left(72 + \frac{20}{3}\right)\right)^{\frac{2}{3}} = ...$

Their work *should* lead to the correct answer so the index work must be correct e.g. $\sqrt{118^3}$ is M0.

$\sqrt[3]{118^2}$ or $118^{\frac{2}{3}}$ are acceptable as values, i.e., the bracket must be evaluated.

May be implied by awrt 24 (cm) following $r^{\frac{3}{2}} = 118$ or by awrt 24.1 (cm). Ignore units for M1.

A1: cao 241mm or 241 or 24.1cm but do not accept 24.1 or e.g. $\sqrt[3]{118^2}$ cm
Correct answer with units implies both marks.

(d)

B1: Examples of acceptable answers (which must relate to the **model in context**):

- (The model will not hold indefinitely as) the balloon may burst/pop
- The balloon is unlikely to be (perfectly) spherical (condone circular)
- The model predicts the balloon will increase in size without limit (which is unrealistic)

Note $t \to \infty \Rightarrow r \to \infty$ is unrealistic / impossible scores B0 unless they reference e.g. the radius.
Condone the presence of additional remarks such as "the balloon may not inflate at the same rate" or "the radius of the balloon might not start at 0" that have already been addressed in the model, but these answers alone score B0.