| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **15(i)** | $k^2 - 4k + 5 = (k-2)^2 + \ldots$ | M1 | 2.1 |
| | $k^2 - 4k + 5 = (k-2)^2 + 1$ so $k^2 - 4k + 5 \ldots 1$<br>so $k^2 - 4k + 5$ is always positive* | A1* | 2.4 |
| | | **(2)** | |

**Notes**

(i) Note: Using e.g. $x$ throughout is acceptable for both marks.

**M1:** Starts the process of showing the given expression is positive.

Completing the square requires $(k-2)^2 \pm \ldots$

Differentiation requires them to differentiate to a linear expression in $k$, set = 0 (which may be implied), solve for $k$ and substitute their $k$ into $k^2 - 4k + 5$ to reach a value. Ignore e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}$

Discriminant requires a calculation of $b^2 - 4ac = (\pm 4)^2 - 4 \times 1 \times 5 \{= -4\}$ and might be seen embedded in the quadratic formula.

Sketches on their own are insufficient without working to find the minimum point algebraically.

Stating the minimum is (2, 1) without any evidence is M0.

**A1*:** Completes the proof with no errors and correct reasoning.

Accept e.g. "hence proved" in place of "so $k^2 - 4k + 5$ is always positive" as long as there is sufficient justification for it being "proved".

As a minimum expect to see e.g.:

- $k^2 - 4k + 5 = (k-2)^2 + 1$ which is always positive as $\underline{(k-2)^2 \ldots 0}$ (or as squares are always positive **or zero**). Must be a correct statement. Do not condone e.g. $(k-2)^2 > 0$
- $k^2 - 4k + 5 = (k-2)^2 + 1$ so (2, 1) is the **minimum** (point) so $k^2 - 4k + 5$ is always positive.
- $k^2 - 4k + 5 = (k-2)^2 + 1$ so $\underline{k = 2 \pm \sqrt{-1}}$ hence $k^2 - 4k + 5$ has **no real roots** and as $\boldsymbol{k^2}$ **has a positive coefficient** (condone e.g. positive $k^2$ or positive quadratic or $a > 0$), hence proved.
- $2k - 4 = 0 \Rightarrow k = 2 \Rightarrow k^2 - 4k + 5 = 1$ is the **minimum** value so $k^2 - 4k + 5$ is always positive.
- $b^2 - 4ac = 16 - 20 \underline{< 0}$ so $k^2 - 4k + 5 = 0$ has **no real roots** and as $\boldsymbol{k^2}$ **has a positive coefficient** (condone e.g. positive $k^2$ or positive quadratic or $a > 0$), hence proved.

Note that stating $k^2 - 4k + 5 = (k-2)^2 + 1 > 0$ alone is not sufficient for the A1*

Similarly, stating $k^2 - 4k + 5 = (k-2)^2 + 1 > 0$ because $(k-2)^2 > 0$ (or $(k-2)^2$ is positive) is incorrect and scores A0.

**SC:** Attempts at completing the square for $k = 2n$ and/or $k = 2n+1$ (or $k = 2n-1$) can score M1 provided at least one is attempted as far as $(\ldots)^2 + \ldots$ e.g. to $(2n-2)^2 + \ldots$ or e.g. $4(n-1)^2 + \ldots$

Candidates are unlikely to complete the argument to score A1 (it is **not** sufficient to just consider odd/even) but if you think they might deserve the A1 then send to Review.

$k = 2n \rightarrow k^2 - 4k + 5 = 4n^2 - 8n + 5$ leading to $(2n-2)^2 + 1$ or $4(n-1)^2 + 1$

$k = 2n+1 \rightarrow k^2 - 4k + 5 = 4n^2 - 4n + 2$ leading to $(2n-1)^2 + 1$ or $4\left(n - \frac{1}{2}\right)^2 + 1$

$k = 2n-1 \rightarrow k^2 - 4k + 5 = 4n^2 - 12n + 10$ leading to $(2n-3)^2 + 1$ or $4\left(n - \frac{3}{2}\right)^2 + 1$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **15(ii)** | Attempts to solve any 1 pair of the relevant 11 sim. equations.<br>e.g. **one of**<br>$\left.\begin{array}{r}3x+2y=28\\2x-5y=1\end{array}\right\}\Rightarrow x=...,(y=...)$<br>**or** … (see notes for all 11) | M1 | 2.1 |
| | Attempts to solve any 2 pairs of the relevant 11 sim. equations with at least one correct and correctly rejected.<br>e.g. **both**<br>$\left.\begin{array}{r}3x+2y=28\\2x-5y=1\end{array}\right\}\Rightarrow x=\frac{142}{19},\left(y=\frac{53}{19}\right)$ Not integers<br>**and**<br>$\left.\begin{array}{r}3x+2y=7\\2x-5y=4\end{array}\right\}\Rightarrow x=...,\ (y=...)$<br>**or** … (see notes for all 11 options) | dM1<br>(A1 on EPEN) | 2.2a |
| | Attempts to solve all 5 pairs of the relevant sim. equations with positive RHS<br>**or** e.g.<br>$\left.\begin{array}{r}3x+2y=28\\2x-5y=1\end{array}\right\}\Rightarrow x=\frac{142}{19},\left(y=\frac{53}{19}\right)$ Not integers<br>**and**<br>$\left.\begin{array}{r}3x+2y=7\\2x-5y=4\end{array}\right\}\Rightarrow x=...,\ (y=...)$<br>**and**<br>all other cases are not possible as $3x + 2y$ □ 5 | ddM1 | 2.1 |
| | Requires:<br>• All cases considered, with correct values and rejected<br>• Correct reasons given in each case e.g. "not integers"<br>• Concluding statement e.g. "hence proven" | A1 | 2.4 |
| | | **(4)** | |
| | | | **(6 marks)** |

**Notes**

15(ii)

**General Note:**

Throughout this question we are condoning if candidates do not reference the following two points which are deemed fairly trivial and acceptable to be omitted at A-level:

- As $x$ and $y$ are integers then both $3x + 2y$ and $2x - 5y$ are integers.
- As $x$ and $y$ are positive then $3x + 2y > 0$

As such, we only *require* candidates to prove that there are no positive integer solutions $x$ and $y$ to simultaneous equations that have a positive RHS.

**Note:** Any attempt to solve the given simultaneous equations does not score any marks on its own.
Any attempts that use substitutions such as $x = 2n$ etc. are unlikely to score any marks.
There are other methods to eliminate some pairs of simultaneous equations e.g. by showing that the only solution to $3x + 2y = 7$ is $(1, 2)$ which is not on $2x - 5y = 4$. Use Review in such cases.

**M1:** Attempt to solve any one of the other 11 possible cases (labelled *A*-*K*) below to find a value for $x$ or a value for $y$. The attempt may be implied by a value for $x$ or $y$ which need not be correct.

**dM1:** Attempts to solve any two of the other 11 possible cases below to find a value for $x$ or a value for $y$ **with** at least one correct **and** correctly rejected. It is not necessary to find both values of $x$ and $y$ unless the correct value found does not cause a contradiction (see cases *D* and *G*).

**ddM1:** Attempts to solve all 5 pairs (*A*-*E*) of the relevant simultaneous equations with positive RHS **or** attempts to solve cases *A* and *B* and justifies that these are the only cases that need checking using either:

- $3x+2y \ldots 5$ (because $x$ and $y$ are positive) **or**
- $3x+2y > 2x-5y$ (because $x$ and $y$ are positive) [cases *F*, *G* and *H* do not need checking using this approach because of the general note]

Their values for $x$ and $y$ do not need to be correct for this mark as long as the dM1 is scored.

**A1:** cso Shows that all necessary cases are impossible with correct values, correct reasons, and a minimal conclusion e.g. "hence proved". There is no need to say "contradiction" or restate the objective (you can also ignore any inaccurate attempt to restate the objective).
All 5 pairs of simultaneous may be rejected in one go if the rejection is sufficiently clear.
The mechanics of solving the simultaneous equations does not need to be shown.

$$A: \left.\begin{array}{r} 3x+2y=28 \\ 2x-5y=1 \end{array}\right\} \Rightarrow x=\frac{142}{19}, \left(y=\frac{53}{19}\right) \quad \text{Not integers}$$

$$B: \left.\begin{array}{r} 3x+2y=7 \\ 2x-5y=4 \end{array}\right\} \Rightarrow x=\frac{43}{19}, \left(y=\frac{2}{19}\right) \quad \text{Not integers}$$

$$C: \left.\begin{array}{r} 3x+2y=4 \\ 2x-5y=7 \end{array}\right\} \Rightarrow x=\frac{34}{19}, \left(y=-\frac{13}{19}\right) \quad \text{Not integers/not positive}$$

$$D: \left.\begin{array}{r} 3x+2y=2 \\ 2x-5y=14 \end{array}\right\} \Rightarrow (x=2),\ y=-2 \quad \text{Not positive}$$

$$E: \left.\begin{array}{r} 3x+2y=1 \\ 2x-5y=28 \end{array}\right\} \Rightarrow x=\frac{61}{19}, \left(y=-\frac{82}{19}\right) \quad \text{Not integers/not positive}$$

$$F: \left.\begin{array}{r} 3x+2y=-1 \\ 2x-5y=-28 \end{array}\right\} \Rightarrow x=-\frac{61}{19}, \left(y=\frac{82}{19}\right) \quad \text{Not integers/not positive}$$

$$G: \left.\begin{array}{r} 3x+2y=-2 \\ 2x-5y=-14 \end{array}\right\} \Rightarrow x=-2, (y=2) \quad \text{Not positive}$$

$$H: \left.\begin{array}{r} 3x+2y=-4 \\ 2x-5y=-7 \end{array}\right\} \Rightarrow x=-\frac{34}{19}, \left(y=\frac{13}{19}\right) \quad \text{Not integers/not positive}$$

$$I: \left.\begin{array}{r} 3x+2y=-7 \\ 2x-5y=-4 \end{array}\right\} \Rightarrow x=-\frac{43}{19}, \left(y=-\frac{2}{19}\right) \quad \text{Not integers/not positive}$$

$$J: \left.\begin{array}{r} 3x+2y=-14 \\ 2x-5y=-2 \end{array}\right\} \Rightarrow x=-\frac{74}{19}, \left(y=-\frac{22}{19}\right) \quad \text{Not integers/not positive}$$

$$K: \left.\begin{array}{r} 3x+2y=-28 \\ 2x-5y=-1 \end{array}\right\} \Rightarrow x=-\frac{142}{19}, \left(y=-\frac{53}{19}\right) \quad \text{Not integers/not positive}$$