13. (a) Given that a is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$$

(4)

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x = k\ a^2$$

where k is a constant to be found.

(4)