Question	Scheme	Marks	AOs
7 (a)	Either $x \le -1$ or $2 \le x \le 5$	M1	2.2a
	Both $\{x : x \in \mathbb{R}, x \le -1\} \cup \{x : x \in \mathbb{R}, 2 \le x \le 5\}$ o.e.	A1	2.5
		(2)	
(b)	States $(y =)\ \alpha(x+1)^2(x-5)^2$ or $(\mathrm{f}(x) =)\ \alpha(x+1)^2(x-5)^2$	M1	1.1b
	Substitutes $(0, -75)$ into $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α	dM1	3.1a
	$y = -3(x+1)^2(x-5)^2$ o.e.	A1	2.1
		(3)	
(c)	Substitutes $x = 2$ into their $y = -3(x+1)^2(x-5)^2 \Rightarrow y = (-243)$	M1	2.1
	$0 < k < 243$	A1ft	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Either

- $x \le -1$ o.e. e.g. $-1 \ge x$
- $2 \le x \le 5$ o.e.

but condone use of strict inequalities anywhere for this mark.
e.g. $2 < x < 5$ or $2 < x \le 5$ or $2 \le x < 5$ May also write e.g. $x < 5$ and $x > 2$ which scores M1 but not "$x < 5$ or $x > 2$"

Allow interval notation such as e.g. $[2,5]$ or $(-\infty, -1]$ or condone e.g. $(2,5)$

Ignore incorrect inequality statements not related to the one which is valid.
e.g. "$2 \leqslant x < 5$ and $x > -1$" which scores M1 for the first inequality.

A1: Requires $\{\ \}$ and $\cup$

$\{x : x \le -1\} \cup \{x : 2 \le x \le 5\}$ or $\{x \mid x \le -1\} \cup \{x \mid 2 \le x \le 5\}$ either way round

but condone $\{x \le -1\} \cup \{2 \le x \le 5\}$, $\{x \le -1 \cup 2 \le x \le 5\}$.

Allow e.g. $\{x : x \le -1\} \cup \{x : 2 \le x \cap x \le 5\}$

Use of $\cap$ to join the two separate regions is A0
It is acceptable (but not required) to mention $\mathbb{R}$
e.g. $\{x : x \in \mathbb{R}, x \le -1\} \cup \{x : x \in \mathbb{R}, 2 \le x \le 5\}$

Condone use of a lower limit written as e.g. $\{x : -\infty \le x \le -1\} \cup \{x : 2 \le x \le 5\}$

(b) **Note a correct equation written down scores all 3 marks.**
A correct expression but missing e.g. $y = ...$ or $f(x) = ...$ scores M1dM1A0

M1: Forms the equation of the form $(y =)\ \alpha(x+1)^2(x-5)^2$. Condone $\alpha = 1$

Award for sight of $\alpha(x+1)^2(x-5)^2$ even with $\alpha = 1$ i.e. $(x+1)^2(x-5)^2$

dM1: Substitutes $(0, -75)$ into the form $y = \alpha(x+1)^2(x-5)^2$ and attempts to find the value for α. It is dependent on the previous method mark.

A1: $y = -3(x+1)^2(x-5)^2$ o.e. (e.g. $y = -3x^4 + 24x^3 - 18x^2 - 120x - 75$)

isw after a correct answer. Condone $f(x) = -3(x+1)^2(x-5)^2$ but not $C = -3(x+1)^2(x-5)^2$

A correct equation scores all 3 marks. Allow if seen in (c)
isw if they attempt to multiply out.

Alternative I part (b):

Using the form $y = ax^4 + bx^3 + cx^2 + dx + e$, then setting up and solving simultaneous equations.

There are various versions of this but can be marked similarly.

M1: Sets e equal to -75 (may just be seen in their equation) **and** forms **three correct** different equations in a, b, c and d which may be unsimplified.

Note that the form $y = ax^4 + bx^3 + cx^2 + dx + e$ is M0 until e is set equal to -75

There are 5 equations that can be formed, only 3 are necessary for this mark.
Do not condone slips.

Using $(-1, 0)$ $\Rightarrow 0 = a - b + c - d - 75$ o.e.

Using $(5, 0)$ $\Rightarrow 0 = 625a + 125b + 25c + 5d - 75$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = 2$ $\Rightarrow 0 = 32a + 12b + 4c + d$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = -1$ $\Rightarrow 0 = -4a + 3b - 2c + d$ o.e.

Using $\frac{dy}{dx} = 0$ at $x = 5$ $\Rightarrow 0 = 500a + 75b + 10c + d$ o.e.

dM1: Forms **four correct** different equations and solves to find values for a, b, c and d. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y = -3x^4 + 24x^3 - 18x^2 - 120x - 75$ o.e. isw if they attempt to factorise but withhold this mark if they e.g. divide all terms by 3.

Condone $f(x) = ...$ but not $C = ...$

A correct equation scores all 3 marks. Allow if seen in (c)

Alternative II part (b): Uses the form $y=(x+1)(x-5)(ax^2+bx+c)$

M1: Substitutes $x=0,\ y=-75 \quad -75=-5c \Rightarrow c=15$, multiplies out, differentiates

$$\Rightarrow \frac{dy}{dx}=(2x-4)(ax^2+bx+15)+(x^2-4x-5)(2ax+b)$$

and forms **a correct equation** in a and b which may be unsimplified.

Using $\frac{dy}{dx}=0$ at $x=2 \qquad \Rightarrow 0=4a+b$ o.e.

Using $\frac{dy}{dx}=0$ at $x=-1 \qquad \Rightarrow 0=a-b+15$ o.e.

Using $\frac{dy}{dx}=0$ at $x=5 \qquad \Rightarrow 0=5a+b+3=0$ o.e.

dM1: Forms **two correct** different equations and solves to find values for a and b. You do not need to be concerned by the process of solving. A calculator can be used to solve the equations.

A1: $y=(x+1)(x-5)(-3x^2+12x+15)$ o.e. isw if they attempt to multiply out or factorise

Condone $f(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

Alternative III part (b): Uses $\frac{dy}{dx}=\beta(x+1)(x-2)(x-5)$ (β may be 1) and integrates.

M1: Integrates $\left(\frac{dy}{dx}=\right)\beta(x+1)(x-2)(x-5)$ to $(y=)\ \beta\left(\frac{1}{4}x^4-2x^3+\frac{3}{2}x^2+10x+k\right)$ and forms **one correct** equation using either $(0,-75)$: $-75=\beta k$ (allow $-75=k$)

$(-1,0)$: $0=\beta\left(\frac{1}{4}+2+\frac{3}{2}-10+k\right) \qquad (5,0)$: $0=\beta\left(\frac{625}{4}-250+\frac{75}{2}+50+k\right)$

dM1: Forms a different equation using one of $(0,-75), (-1,0), (5, 0)$ and solves to find values for β and k. You do not need to be concerned by the process of solving . A calculator can be used to solve the equations.

A1: $y=-12\left(\frac{1}{4}x^4-2x^3+\frac{3}{2}x^2+10x+\frac{25}{4}\right)$ o.e. isw if they attempt to multiply out or factorise

Condone $f(x)=...$ but not $C=...$ but withhold this mark if they e.g. divide all terms by 3. A correct equation scores all 3 marks. Allow if seen in (c)

(c)

M1: Substitutes $x=2$ into their $y=-3(x+1)^2(x-5)^2$ (must be a quartic in any form) and proceeds to find a value for y. Sight of their $\pm y$ (or ± 243) scores M1.
You may need to check this on your calculator if only a value is seen.

A1ft: $0<k<243$ o.e. ft on their negative y value at $x=2$.
Allow use of set notation, interval notation and allow e.g. $k<243,\ k>0$ but do not allow OR or $\cup$. Do not accept $0\le k\le 243$ o.e.
If there are multiple attempts at describing the region, mark what appears to be their final answer.
This mark can only be scored if they have a negative quartic graph function
i.e. $\alpha<0$ for their $y=\alpha(x+1)^2(x-5)^2$ or $a<0$ for their $y=ax^4+bx^3+cx^2+dx+e$