Question	Scheme	Marks	AOs
10 (a)	Attempts to add $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$	M1	1.1b
	$\left(\overrightarrow{PR} =\right)\ 8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$	A1	1.1b
		(2)	
(b)	Attempts to show the triangle is isosceles (**or** right-angled) e.g. Attempts $\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2 + 8^2 + (-2)^2}$ and $\left\|\overrightarrow{QR}\right\| = \sqrt{6^2 + 6^2}$	M1	3.1a
	Shows e.g. $\left\|\overrightarrow{PQ}\right\| = \left\|\overrightarrow{QR}\right\| = \sqrt{72}\ (= 6\sqrt{2})$	A1	1.1b
	Attempts to show the triangle is isosceles **and** right-angled e.g. attempts to find the lengths of all three sides AND e.g. attempts to compare lengths via use of "$a^2 + b^2 = c^2$"	M1	1.1b
	e.g. Shows that $\left\|\overrightarrow{PQ}\right\|^2 + \left\|\overrightarrow{QR}\right\|^2 = \left\|\overrightarrow{PR}\right\|^2$ as $72 + 72 = 144$ so *PQR* is a right-angled triangle	A1	2.1
		(4)	
			(6 marks)

Notes:

(a) If part (a) is not attempted and the correct $\overrightarrow{PR}$ is seen in part (b) then M1A1 can be awarded

M1: Attempts to add $\overrightarrow{PQ} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{QR} = 6\mathbf{i} + 6\mathbf{k}$ with at least one correct component of $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$. A typical misread of $\overrightarrow{QR}$ as $6\mathbf{i} + 6\mathbf{j}$ can score for at least one correct component of $8\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$

A1: Correct vector. Allow $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ but **not** $\begin{pmatrix} 8\mathbf{i} \\ 8\mathbf{j} \\ 4\mathbf{k} \end{pmatrix}$ and **not** (8, 8, 4)

Condone $\begin{matrix} 8 \\ 8 \\ 4 \end{matrix}$ for $\begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix}$ Do not apply isw here but award for e.g. $8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = \begin{pmatrix} 8\mathrm{i} \\ 8\mathrm{j} \\ 4\mathrm{k} \end{pmatrix}$

E.g. if they obtain $\overrightarrow{PR} = 8\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$ and then say $\overrightarrow{PR} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ then award A0

(b) **Note that M1A0M1A1 is not possible. If they have an incorrect vector in part (a) then the maximum score is M1A1M1A0. A misread of $\overrightarrow{QR}$ as $6\mathbf{i} + 6\mathbf{j}$ in (b) can only score a maximum M1A1M1A0**

They may attempt to show the triangle is either isosceles or right-angled in either order. You will need to look through their solution and award the order which scores most marks. Usually it will be isosceles first. Condone slips to be recovered.

To show the triangle is isosceles they only need to show **two sides** (or **two angles**) are the same. They do not need to consider the other side to show it is isosceles.

M1: Attempts to show that the triangle is either isosceles (**or** right-angled). See table below.

A1: Fully shows that the triangle is isosceles (or right-angled). See table below. Allow slips in their method if recovered as long as they proceed to correct lengths or values. **A conclusion that the triangle is isosceles or right-angled is not required for this mark**.

Isosceles	**Requirement for M1 examples**	**Requirement for A1 examples**
Using lengths	Attempts to find the length or length^2 of PQ and QR: $\left(\left\|\overrightarrow{PQ}\right\|=\right)\sqrt{2^2+8^2+(-2)^2}\ (=\sqrt{72}=6\sqrt{2})$ or seen as e.g. $2\sqrt{1^2+4^2+(-1)^2}\ (=2\sqrt{18})$ $\left(\left\|\overrightarrow{QR}\right\|=\right)\sqrt{6^2+6^2}\ (=\sqrt{72}=6\sqrt{2})$ or may be seen as e.g. $3\sqrt{2^2+2^2}\ (=3\sqrt{8})$ May be implied by e.g. $6\sqrt{2}$ Condone missing brackets around $(-2)^2$ provided the intention is clear to square and add implied by e.g. $6\sqrt{2}$	States or shows that $\left\|\overrightarrow{PQ}\right\|=\left\|\overrightarrow{QR}\right\|\left(=\sqrt{72}\,(=6\sqrt{2})\right)$, or equivalent. Accept e.g. $PQ^2=QR^2$ or "both are 72" Condone poor notation and/or labelling of lengths provided they are not clearly referring to the longest length. e.g. achieves $6\sqrt{2}$ for both PQ and QR and states they are the same scores M1A1 **Only stating isosceles without a comparison of *PQ* and *QR* is A0**
		Uses the sine rule with the lengths and angles embedded in the correct places e.g. $\dfrac{\sin\angle QPR}{QR}=\dfrac{\sin\angle PRQ}{PQ}$ Deduces $\sin\angle QPR=\sin\angle PRQ$ so the angles are the same o.e.
Right-angled	**Requirement for M1 examples**	**Requirement for A1 examples**
Using lengths	Attempts to find all three lengths or lengths^2 $\left(\left\|\overrightarrow{PR}\right\|=\right)\sqrt{"8"^2+"8"^2+"4"^2}\ \ (=12)$ or may be seen as e.g. $4\sqrt{"2"^2+"2"^2+"1"^2}$ or implied by their 12 **AND ATTEMPTS** $\left(PQ^2+QR^2=PR^2\Rightarrow\right)"72"+"72"="144"$	States or shows that $\|PQ\|^2+\|QR\|^2=\|PR\|^2$, or equivalent Condone poor notation and/or labelling of lengths.
	Attempts to find all three lengths or lengths^2 (see above for guidance) **AND ATTEMPTS** the cosine rule correctly to find $\cos PQR=\dfrac{"72"+"72"-"144"}{2\times"\sqrt{72}"\times"\sqrt{72}"}$ o.e.	States or shows $\cos PQR=0$
Scalar dot product (Further Maths)	Attempts e.g. $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix}=(2\times6)+(8\times0)+(-2\times6)$ oe **Must see the calculation for M1**	States or shows that $(2\times6)+(8\times0)+(-2\times6)=0$ oe (they do not need to write anything more for this mark)
	$\left(\cos PQR=\right)\dfrac{(2\times6)+(8\times0)+(-2\times6)}{"\sqrt{72}"\times"\sqrt{72}"}$ oe	$\cos PQR=\dfrac{(2\times6)+(8\times0)+(-2\times6)}{\sqrt{72}\times\sqrt{72}}=0$

M1: Attempts to show that the triangle is both isosceles **and** right-angled. Usually they will have shown the triangle is isosceles and attempt to show that it is right-angled. **Here are some examples but there will be others**:

Right-angled	**Method examples (required for second method mark)**
Using lengths	Attempts to find all three lengths or lengths^2 **AND ATTEMPTS** $\left(PQ^2+QR^2=PR^2 \Rightarrow\right)$ "72"+"72"="144"
	Attempts to find all three lengths or lengths^2 (see above for guidance) **AND ATTEMPTS** the cosine rule in an attempt to find $\cos PQR = \dfrac{\text{"72"}+\text{"72"}-\text{"144"}}{2\times\text{"}\sqrt{72}\text{"}\times\text{"}\sqrt{72}\text{"}}$
Scalar dot product	Attempts $\begin{pmatrix}2\\8\\-2\end{pmatrix}\bullet\begin{pmatrix}6\\0\\6\end{pmatrix}=(2\times6)+(8\times0)+(-2\times6)$
	$\left(\cos PQR=\right)\dfrac{(2\times6)+(8\times0)+(-2\times6)}{\text{"}\sqrt{72}\text{"}\times\text{"}\sqrt{72}\text{"}}$ oe

Alternatively, having already shown the triangle is right-angled they may:

- use trigonometry to show that the two base angles are both $45°$
- attempt to find the required lengths or lengths^2 of *PQ* and *PR* if not already found
  (Note they may also find the length or length^2 of *QR* and may use the sine rule or cosine rule)

Note if they have shown either property then we will condone making an assumption of the other property to justify the size of angle *PRQ* and/or angle *QPR* and may use trigonometry to show either

$$\left(\sin\theta=\frac{6\sqrt{2}}{12}\Rightarrow\right)\theta=\arcsin\left(\frac{6\sqrt{2}}{12}\right)=45°$$ (we must see arcsin or $\sin^{-1}$)

$$\sin\theta=\frac{6\sqrt{2}}{12}\Rightarrow\sin\theta=\frac{\sqrt{2}}{2}\left(\text{or }\frac{1}{\sqrt{2}}\right)\Rightarrow\theta=45°$$ (since this is a known angle)

$$\sin 45°=\frac{\sqrt{2}}{2}\left(\text{or }\frac{1}{\sqrt{2}}\right)=\frac{6\sqrt{2}}{12}$$ (This requires all 3 equivalences (can be in any order))

Using trigonometry (SOHCAHTOA) to show the second property they will e.g.

- Use triangle *PQR* and assume right angled
- Split triangle *PQR* into two right angled congruent triangles using the isosceles property

If they use triangle *PQR* then it requires

- For M1: Calculations showing the required property to score M1
- For A1: To either draw a labelled right angled triangle or state the assumption that it is right angled and conclude

If they split the triangle *PQR* in half then they have formed two right angled congruent triangles which (provided they had two lengths the same already) will not be an assumption of a right angle. **Then it requires**

- For M1: Calculations to find either at least one of the base angles of 45° OR to find the right angle. It should be clear whether they
  - found a base angle of PQR and doubled it or
  - found half of the right angle and doubled it.
- For A1: Fully correct calculations and conclusion

Note if they find a 45 degree angle and double it then it needs to be clear whether this is the right angle or if it is the sum of two base angles because there are four angles of size 45 degrees via this method.
They would have to e.g. mention about the angle sum of a triangle oe or show a clearly labelled diagram and calculations with labels that match the diagram.

Note that use of tan is unlikely to score because if they just use two equal lengths for the two shorter sides of their right angled triangle, then tan(A)=1, so the angles will always be 45°, 45° and 90° – it is the use of the length of 12 (or 6) which is going to lead to showing the triangle is right angled via these routes.

A1: Fully shows that the triangle is isosceles and right-angled **and concludes** that the triangle is **both isosceles and right-angled**. These conclusions may appear at separate stages of their solution. Condone poor notation and/or labelling of lengths provided the intention is clear. Condone slips if recovered. If they have a preamble then they must have a minimal conclusion e.g. proven, tick, QED

Note if they have an incorrect vector in part (a) then the maximum score is M1A1M1A0