Question	Scheme	Marks	AOs
11 (a)	Uses $t=0, V=20\,000 \Rightarrow 20\,000 = 1500 + A\mathrm{e}^{0}$	M1	3.4
	$A = 18500$	A1	1.1b
	$t=2.5, V=12\,000 \Rightarrow 12\,000 = 1500 + 18\,500\mathrm{e}^{-k\times 2.5}$ $\Rightarrow 10500 = 18500\mathrm{e}^{-k\times 2.5} \Rightarrow k = ...\left(= -\frac{2}{5}\ln\frac{21}{37} = \text{awrt } 0.227\right)$	dM1	3.1b
	$V = 1500 + 18500\,\mathrm{e}^{-0.227t}$	A1	3.3
		(4)	
(b)	Achieves $\left(\frac{\mathrm{d}V}{\mathrm{d}t} =\right) -kA\mathrm{e}^{-kt}$ or $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	B1	1.1b
	Substitutes $A\mathrm{e}^{-kt} = V - 1500$ into $\left(\frac{\mathrm{d}V}{\mathrm{d}t} =\right) -kA\mathrm{e}^{-kt}$ or $"18500"\mathrm{e}^{"-0.227"t} = V - 1500$ into $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$	M1	3.4
	Rate of change in value of car is $\left(\frac{\mathrm{d}V}{\mathrm{d}t} =\right) -k(V-1500)$ or $"-0.227"(V-1500)$ *	A1*	2.1
		(3)	
(c)	Suggests a suitable limitation of the model (see notes)	B1	3.5b
		(1)	
			(8 marks)

Notes:

Mark (a) and (b) together

(a)

M1: Uses the equation of the model with $t=0, V=20\,000 \Rightarrow 20000 = 1500 + A\mathrm{e}^{0}$ o.e.
May be implied by 18500

A1: $A = 18500$ (18500 with no working seen scores M1A1) Ignore £ if present.

dM1: Attempts to use the equation of the model $t=2.5, V=12\,000$
$\Rightarrow 12000 = 1500 + "18500"\mathrm{e}^{-2.5k}$ and proceeds to $C\mathrm{e}^{\pm k\times 2.5} = D$ (where $C\times D > 0$ and allow $C=1$) **before proceeding** to find a value for *k*. Allow them to have a non-numerical *C* for this mark.
Note it cannot be implied by their awrt ± 0.227 so
$12000 = 1500 + "18500"\mathrm{e}^{-2.5k} \Rightarrow k = 0.227$ scores dM0A0 as we need to see the intermediate stage $C\mathrm{e}^{\pm k\times 2.5} = D$. (typically look for $18\,500\mathrm{e}^{-k\times 2.5} = 10\,500 \Rightarrow k = ...$ or condone to be implied by a correct expression involving logarithms for their *A*)
It is dependent on the previous method mark.

A1: $V = 1500 + 18500\,\mathrm{e}^{\text{awrt} -0.227t}$ o.e. e.g. $t = \dfrac{\ln\left(\dfrac{V-1500}{18500}\right)}{\text{awrt} -0.227}$ Allow *k* to be exact.

(b)

B1: Differentiates to a form $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right)-kA\mathrm{e}^{-kt}$ where k and A may be their values from (a)

e.g. $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}$ May just see e.g. (using an exact k) $"-4191.3..."\mathrm{e}^{"-0.227"t}$ or e.g. (using k to 3sf) $"-4199.5"\mathrm{e}^{"-0.227"t}$ but do not be too concerned by over rounding provided the intention is clear that it is their $-kA$

Do not be too concerned by the left hand side / poor labelling of the derivative.

M1: Substitutes $A\mathrm{e}^{-kt}=V-1500$ into their $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right)\pm kA\mathrm{e}^{-kt}$ to form an expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ in terms of V. May see e.g. $"-0.227"\times"18500"\mathrm{e}^{"-0.227"t}\Rightarrow"-0.227"(V-1500)$.

Condone recovery of a sign slip on the index on $\mathrm{e}^{-kt}\rightarrow\mathrm{e}^{kt}$ provided it is before they substitute $A\mathrm{e}^{-kt}=V-1500$ for this mark.

Using the given answer: substitutes $V-1500=A\mathrm{e}^{-kt}$ and shows that $-k(V-1500)=-kA\mathrm{e}^{-kt}$ (may be in terms of their A and k)

A1*: Full and complete proof with sight of $\frac{\mathrm{d}V}{\mathrm{d}t}$ oe seen somewhere in their solution and no errors seen. Must see $-kA\mathrm{e}^{-kt}$ (or using their values) before proceeding to the given answer which may be written using their numerical value for k

Using the given answer they must conclude that $\frac{\mathrm{d}V}{\mathrm{d}t}=-k(V-1500)$ which may be written using their numerical value for k

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Alt (b) Separating the variables – may be in terms of their numerical value for *k*

B1ft: Separates the variables correctly and integrates to $\ln(V-1500)=-kt$ with or without $+c$

$$\int\frac{1}{(V-1500)}\mathrm{d}V=\int-k\,\mathrm{d}t\Rightarrow\ln(V-1500)=-kt\ \ (+c)\ \ \text{oe}$$

M1: Proceeds from $...\ln(V-1500)=...kt+c$ o.e. and rearranges to make V the subject. Condone slips.

e.g. $\ln(V-1500)=-kt+c\Rightarrow V-1500=A\mathrm{e}^{-kt}\Rightarrow V=1500+A\mathrm{e}^{-kt}$ or $V=1500+\mathrm{e}^{-kt+c}$ (just look for proceeding to V=… for this mark though.) May be in terms of their numerical values for k and A

A1*: Achieves $V=1500+A\mathrm{e}^{-kt}$ with no errors seen and concludes that $\frac{\mathrm{d}V}{\mathrm{d}t}=-k\left(V-1500\right)$

May be in terms of their numerical values for k and A

Alt (b) Rearranging to make *t* the subject

B1ft: For a correct rearrangement to $t=-\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right)$ ft on their k

M1: Differentiates $...\ln(V-1500)$ to $\frac{...}{V-1500}$ and then finding $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{\left(\frac{\mathrm{d}t}{\mathrm{d}V}\right)}$

Typically look for $t=-\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right)\Rightarrow\frac{\mathrm{d}t}{\mathrm{d}V}=-\frac{1}{k}\left(\frac{1}{V-1500}\right)$ o.e. so they may have an unsimplified version of this e.g.

$$t=-\frac{1}{k}\ln\left(\frac{V-1500}{18500}\right)\Rightarrow\frac{\mathrm{d}t}{\mathrm{d}V}=-\frac{1}{k}\left(\frac{\frac{1}{18500}}{\frac{V-1500}{18500}}\right)\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}t}=-k\left(\frac{\frac{V-1500}{18500}}{\frac{1}{18500}}\right)$$

May be in terms of their numerical *k*. Condone slips.

A1*: Achieves the given answer with no errors and $\frac{\mathrm{d}V}{\mathrm{d}t}$ seen somewhere in their solution

(c) Note the question asks for a limitation of the model. If there is ambiguity over whether the response is referring to the model then try putting "the model suggests" or "the model" in front of their comment to see if this is a valid limitation.
Ignore comments which do not contradict a valid limitation.
If values are given then they must be correct and if it is the value of the car then it must have units (£ or pounds)

B1: **Suitable limitations in context referring to the limitation of the model which score B1**
e.g. (the model suggests) the value/price of the car will never go below £1500
e.g. (the model suggests) the rate of decrease of the value of the car is proportionally the same each year
e.g. (the model suggests) after a certain period of time the car will no longer lose value (condone this property of the model that it will tend to a limit)
e.g. (the model) only takes account of age
e.g. (the model) does not take into account damage / alteration to the car/ mileage
e.g. (the model) predicts that the car's value will always go down
e.g. after many years the car may become worthless whereas the model does not allow for this (valid limitation comparing the car to the model)
e.g. the value of the car may go up where as it only decreases according to the model

Do not accept vague/incorrect/irrelevant or non-contextual comments which score B0
e.g. after many years the car may become worthless
e.g. the value of the car may increase
e.g. damage or alterations to the car may impact the value
e.g. the car will still have value when it is very old (should refer to £1500)
e.g. (the model suggests) the minimum value is 1500 (no units for money)
e.g. (the model suggests) the (value of the) car cannot be negative
e.g. car value will fluctuate which the model will not show (a model is not for this purpose)
e.g. the rate of decrease is proportionally the same each year (no context)